

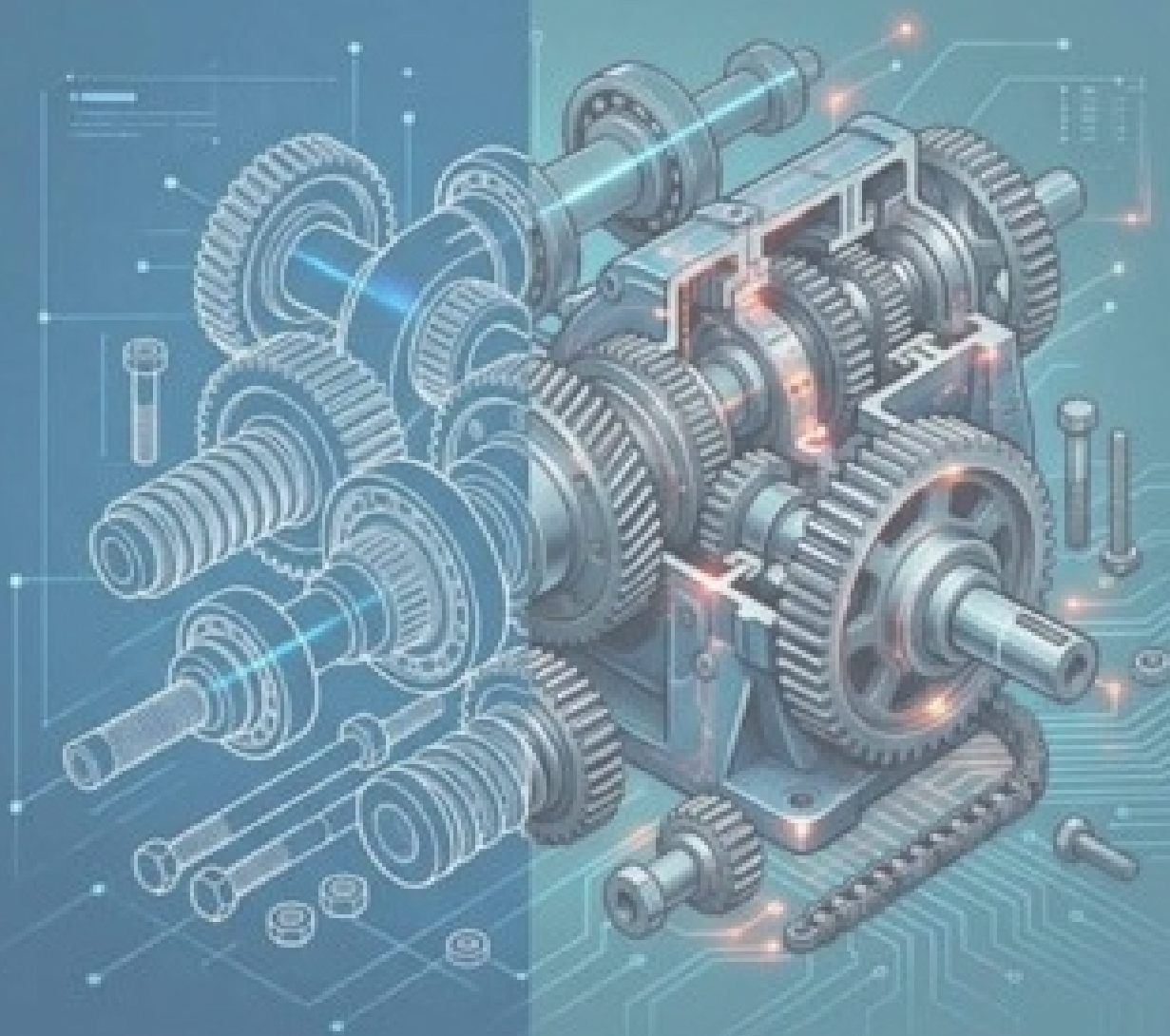
George-Horea CRIȘAN

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Marius PUSTAN

MACHINE ELEMENTS

COURSEBOOK



U.T. PRESS

Cluj-Napoca, 2026

ISBN 978-606-737-867-2

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Preface

A thorough understanding of the essential components of machines represents a crucial stage in the training of future engineers, equipping them to meet the current challenges in mechanical and industrial engineering. This volume has been developed as a course companion for the subject of Machine Elements, and it is primarily intended for undergraduate students specializing in Mechanical and Industrial Engineering. Nevertheless, it may also prove highly valuable to those pursuing master's or doctoral studies in these fields.

This work provides a contemporary approach that supports learning during lectures, project work, laboratory sessions, and individual study. Its content is structured to offer a comprehensive and practical understanding of machine elements, grounding theoretical knowledge in relevant case studies and practical examples. Furthermore, the material is designed to foster critical thinking and problem-solving skills, both of which are essential for the development of engineering professionals. In this way, the present work encourages an integrated and interdisciplinary perspective on the subject matter, in response to the complexity and diversity of the information presented. Consequently, students will be able to gain a deeper insight into the context and significance of each concept explored.

We thus express our hope that this volume will facilitate both active learning in the classroom and thorough preparation for examinations and engineering challenges. This book aims to support the achievement of the fundamental objective that success in engineering depends not only on the accumulation of knowledge but also on the ability to apply it in a creative and effective manner.

We encourage you to approach this course material with curiosity and openness, and to make full use of the resources it offers to enhance the skills necessary for your professional development.

[The Authors]

They are among the most widely used types of assemblies used in the machine building industry (over 60% of a machine's components contain threads).

- the threaded assembly is the most common removable assembly;
- the screw is a seemingly simple machine component, but it involves complicated issues;
- a threaded assembly comprises a component with an external thread called a screw and a component with an internal thread called a nut;
- the main element is the thread.

[illegible]

The thread is a helical ridge generated by the sliding of a profile (geometric shapes - triangle, trapezoid, saw) along a helix wrapped around a cylinder or a cone. The terminology of the thread, according to SR ISO 68-1, is presented in Figure 11.

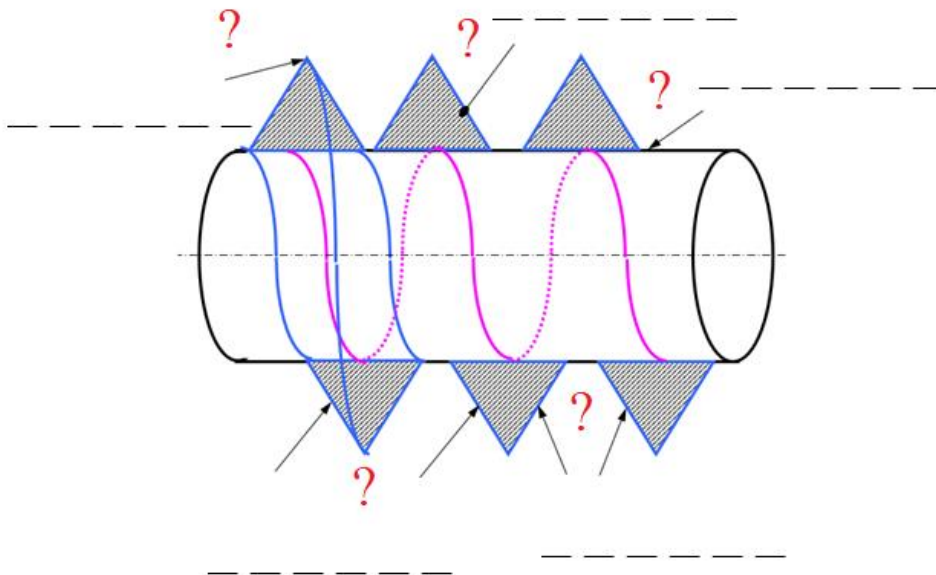


Fig. 1.1 Terminology used for threads– SR ISO 68-1

The main types of threads (shown in Figure 1.2) are:

- a).....;
- b).....;
- c).....;
- d).....

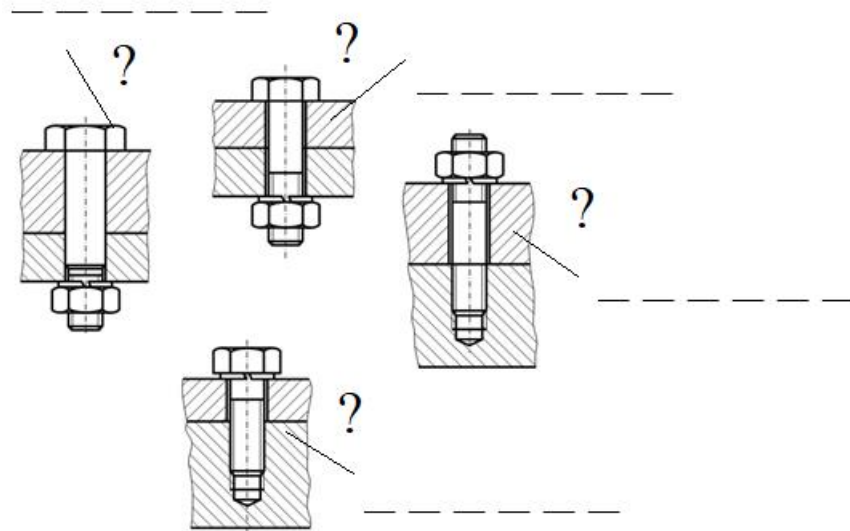


Fig. 1.2 Main types of threads used

In a threaded assembly of screw-nut type, the geometric elements of the thread are presented in figure 1.3 a), while in figure 1.3 b) the lead helix can be observed.

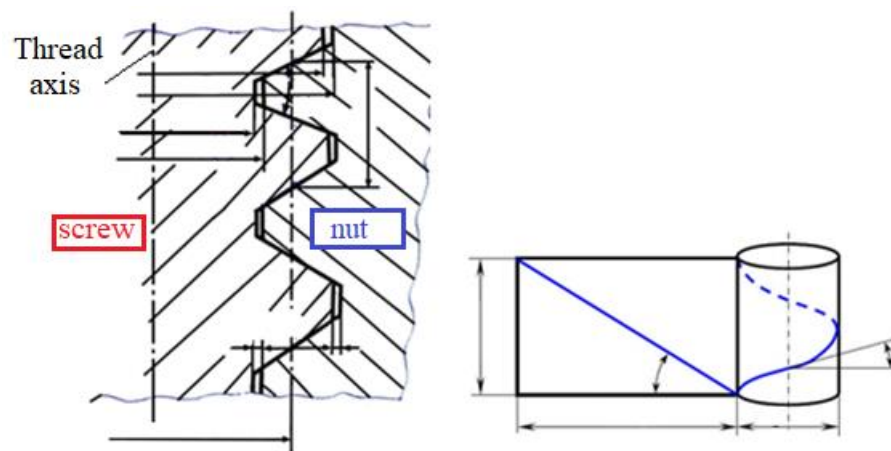


Fig. 1.3 a) Bolt-nut assembly

b) Lead helix

The thread profile – is the outline obtained by intersecting the thread with an axial plane;

The thread flank – is the surface generated by one of the lateral edges of the profile, used to bear the load in the assembly;

Counterpart flanks are flanks generated by the same profile line, participating in bearing the load in the assembly;

The thread crest – is the portion generated by the top of the profile.;

The thread root – is the portion generated by the bottom of the cutting tool;

Pitch – is the distance measured in a direction parallel to the axis of the threaded part, between two points on successive homologous flanks. It is denoted as P and is standardized.;

Number of starts 'n'.

In figure 1.4, the schematic representations of the thread winding directions and its pitch relative to the number of starts are depicted.

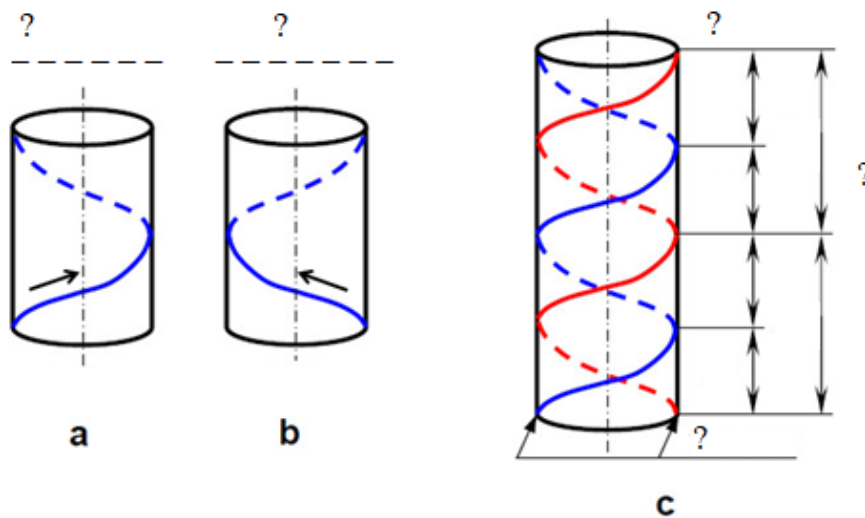


Fig. 1.4 The thread direction and the number of starts

For threads with n starts, the helix pitch P_h is highlighted.

The relationship between pitch P and helix pitch (lead) P_h is: $P_h = n \cdot P$.

The thread coil – is the portion that corresponds to one complete turn of the lead helix P_h .

α –; α_1 –

d –

a_c –

H_1 –

β_2 -

.....

$$\operatorname{tg} \beta_2 = \frac{P}{\pi \cdot d_2}$$

$d_{1(3)}$ –

D –

$d_2 = D_2$ –

$D_{1(3)}$ –

$$\operatorname{tg} \beta_2 = \frac{P_h}{\pi \cdot d_2} = \frac{n \cdot P}{\pi \cdot d_2} \Rightarrow P_h = \pi \cdot d_2 \cdot \operatorname{tg} \beta_2$$

Criteria classification of threads

1. The thread profile

–

–

–

-
-;
 -;

2. *Thread pitch fineness/precision (for the same major diameter d)*

-;
-;
-;

3. *Number of starts*

-;
-;

4. *Direction of the lead helix winding*

-;
-;

5. *Functional role*

-;
 -;
 -;
 -;
 -;
 -;
-

Thread profile and flank angles

In figure 1.5, various types of thread profiles are presented.

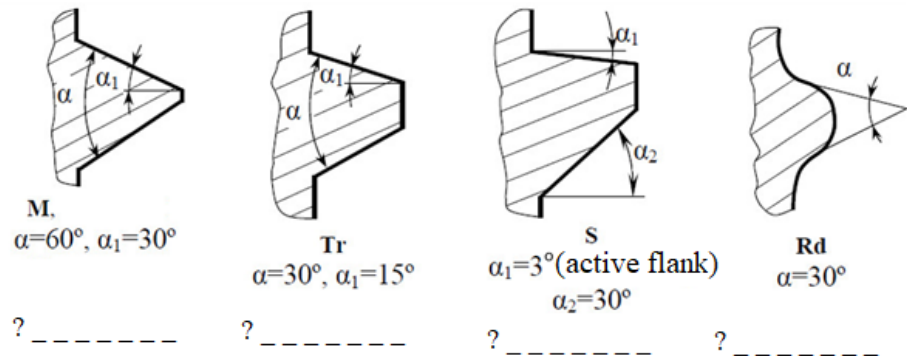


Fig. 1.5 The thread profiles and the profile and flank angles

Notation of general-purpose threads according to SR ISO 261

In figure 1.6, the notation scheme for general-purpose threads is presented.

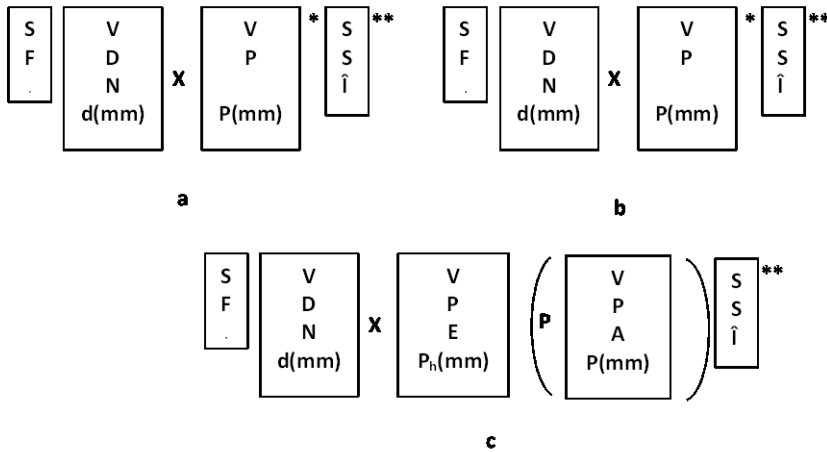


Fig. 1.6 The notation scheme for general-purpose threads.

The legend for figure 1.6.

- a) Metric threads (M) with a single start;
- b) Tr and S threads with a single start;
- c) Multiple-start threads.

SF -;

VDN -;

VP -;

VPE -;

VPA -;

SSÎ -;

* In the case of metric threads, if it is a standard pitch thread, the pitch value is not indicated.;

** the symbol RH (right-hand) is implicit and therefore is not specified unless there is also a left-hand thread on the part..

Materials used in threaded assemblies and their markings

Low carbon construction steels SR EN 10025: S 235, S 275, E 295, E 355 - these steels are used because they provide the best formability.

Non-alloy steel EN 10083-2: 1C25, 1C35, 1C45.

The symbols for the commonly used mechanical property classes for screws are: 3.6; 4.6; 4.8; 5.6; 5.8; 6.8; 8.8; 10.9; 12.9.

Forces and moments in threaded assemblies

The resistances to rotation due to friction between the threads (T_1) and, on the other hand, those generated by the friction between the nut and

washer (bearing part) (T_2) are determined. The following are the expressions for T_1 and T_2 .

Hypothesis for calculation:

For understanding the problem and considering the relative movements between the screw and the nut, tightening the nut under the load $\mathbf{F}$ can be replaced by lifting a body $\mathbf{P}$ with weight $\mathbf{F}$ on the screw thread (as shown in figure 1.7), where the movement occurs along the mean helix (inclined at angle β_2). The nut is rotated using a wrench. At the thread level, a tangential force H is generated, acting at a distance of $\frac{1}{2} d_2$ from the axis.

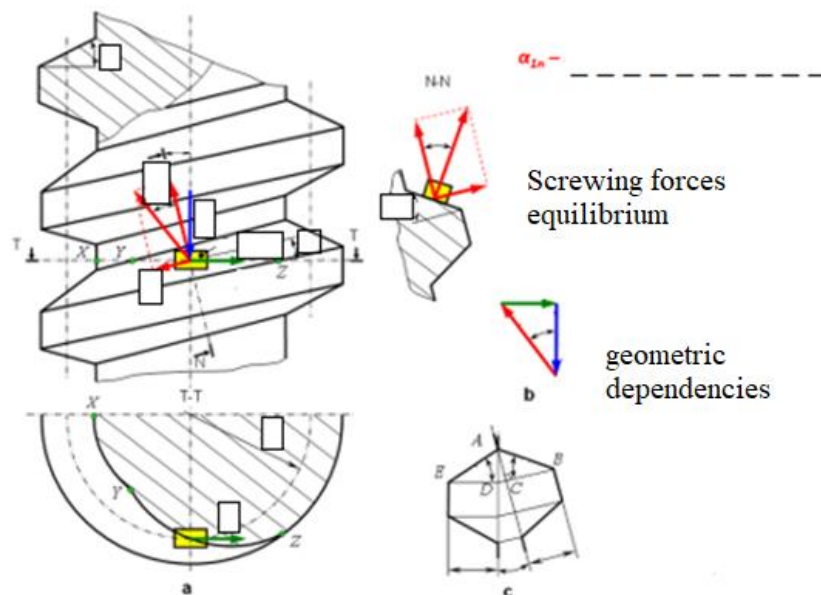


Fig. 1.7 The forces acting on the "body P"

According to figure 1.7, the following forces act on body P:

- F -
- H -

-
- F_{n1} -
 - F_f -
 - R -

From the force equilibrium, the expression for the tangential force can be derived:

$$H = F \cdot \operatorname{tg}(\beta_2 + \phi')$$

- The expression for the torsion torque required to overcome the friction between the threads – screwing torque:

$$T_1 = H \cdot \frac{d_2}{2} = \frac{1}{2} \cdot F \cdot d_2 \cdot \operatorname{tg}(\beta_2 + \phi')$$

where:

- β_2 - helix angle;
- ϕ' – reduced angle of friction.

$$\beta_2 = \arctan\left(\frac{P}{\pi \cdot d_2}\right),$$

Self-locking condition:

$$H' = F \cdot \operatorname{tg}(\phi' - \beta_2)$$

The assembly is self-locking when it does not loosen without the intervention of a tangential force. For unscrewing, a tangential force H' is required (figure 1.8).

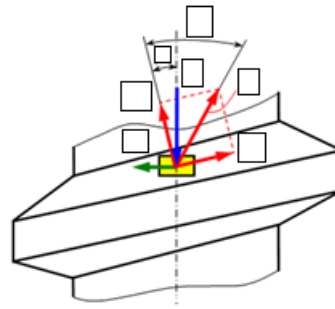


Fig. 1.8 Self-locking condition

Because H' must have some value, thus $H' > 0$, and $F > 0$, and considering that the tangent function is increasing in the first quadrant, the condition is obtained:

$$\phi' - \beta_2 > 0 \Leftrightarrow \beta_2 < \phi'$$

Self-locking condition:

At the limit:: $\beta_2 \leq \phi'$

From the figure, it follows:

$$\operatorname{tg} \phi' = \frac{F_f}{F_0} = \frac{\mu \cdot F_{n1}}{F_{n1} \cdot \cos \alpha_{1n}} = \frac{\mu}{\cos \alpha_{1n}} = \mu', \text{ and } \mu' - \text{apparent coefficient of}$$

friction.

The coefficient of friction depends on:

-
-
-
-

For steel/steel:

-;
-;
-

Determining the frictional torque in the support area

This is a resisting moment generated by the friction between the nut and the washer and is conventionally denoted by the symbol T_2 . It is considered as an example, a bearing extracting press (figure 1.9).

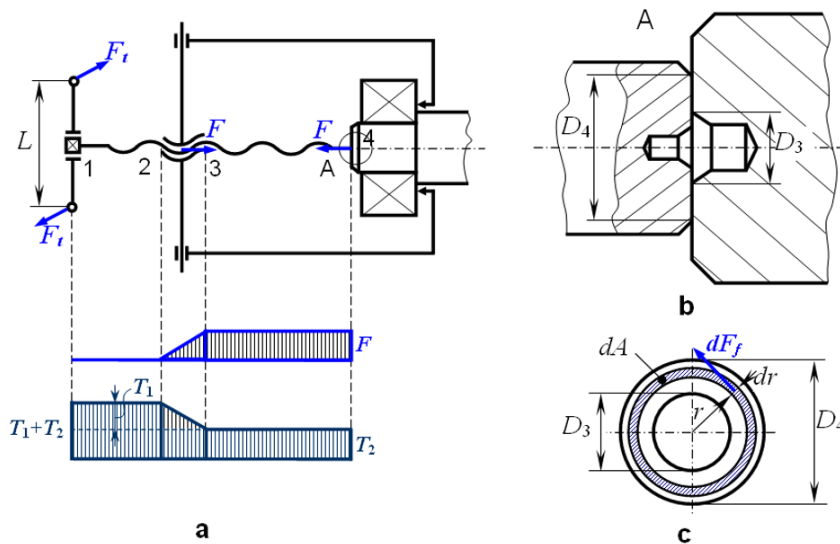


Fig. 1.9 Bearing extracting press: a) Sketch of loads; b) Support detail; c)

Isolated element

Observations/Hypothesis

-
-
-

.....

$$p = \frac{F}{\frac{\pi \cdot (D_4^2 - D_3^2)}{4}},$$

If the contact pressure is determined as follows:

then the friction torque at the support is determined by the following

expression:
$$T_2 = 2\mu \cdot \pi \cdot \frac{1}{3} \cdot \frac{4F}{\pi \cdot (D_4^2 - D_3^2)} \cdot \frac{D_4^3 - D_3^3}{8} \Rightarrow T_2 = \frac{1}{3} \mu \cdot F \cdot \frac{D_4^3 - D_3^3}{D_4^2 - D_3^2}$$

The total friction torque (both from the coils and from the support area) is:

$$T = T_1 + T_2 = \frac{1}{2} \cdot F \cdot d_2 \cdot \operatorname{tg}(\beta_2 + \phi) + \frac{1}{3} \cdot \mu \cdot F \cdot \frac{D_4^3 - D_3^3}{D_4^2 - D_3^2}.$$

Knowing T_1 and T_2 , the actuating element of the bearing extractor can be dimensioned.

$$T = F_t \cdot L \quad \text{respectiv: } L = \frac{T}{F_t} [\text{mm}],$$

where F_t - approximately 150 [N].

The efficiency of threaded assemblies

Efficiency of the screw. For one full rotation of the nut around the screw, the axial displacement of the load occurs by one pitch.

The efficiency of the threaded assembly is: $\eta = \frac{L_u}{L_c}$, where:

L_u – Effective work;

L_c – Mechanical work done.

Calculation hypotheses:

.....

.....

The machining efficiency is consistent throughout the entire length of the thread.

$$\eta = \frac{L_u}{L_c} = \frac{F \cdot p}{H \cdot \pi \cdot d_2} = \frac{F \cdot p}{F \cdot \operatorname{tg}(\beta_2 + \phi') \cdot \pi \cdot d_2} = \frac{\operatorname{tg} \beta_2}{\operatorname{tg}(\beta_2 + \phi')}.$$

For self-locking screws, at the limit $\beta_2 = \phi'$ it is achieved:

$$\eta = \frac{\operatorname{tg} \beta_2}{\operatorname{tg} 2\beta_2} = \frac{\operatorname{tg} \beta_2 \cdot (1 - \operatorname{tg}^2 \beta_2)}{2\operatorname{tg} \beta_2} = \frac{1}{2} - \frac{\operatorname{tg}^2 \beta_2}{2} < \frac{1}{2} = 50\%$$

At the same value of the angle β_2 , the efficiency increase can be achieved by:

.....

.....

.....

Stresses in threads

The main stresses that occur in the threads are:

.....

However, the primary cause of threaded assembly failure is the wear of the threads due to their **crushing** (Figure 1.10).

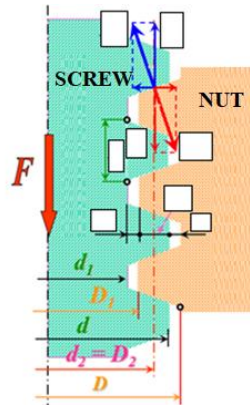


Fig. 1.10 Forces in assembly threads

Thread Crushing (contact pressure)

Simplifying hypotheses: the force is uniformly distributed across all z threads, and each thread is considered a ring (Figure 1.11).

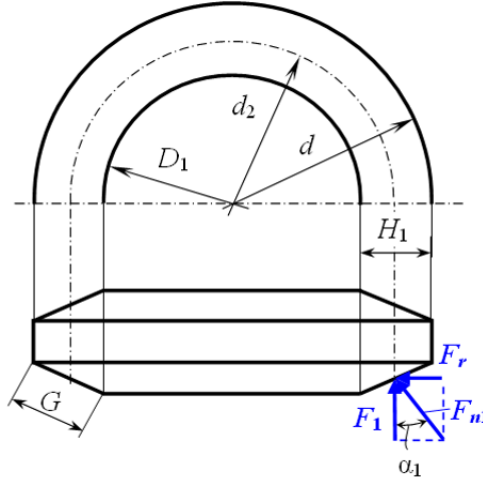


Fig. 1.11 Distribution of forces at the level of a coil

The relationship used for crush checking (contact pressure):

$$q = \frac{F_{n1}}{A_{cone}} = \frac{\frac{F_1}{\cos \alpha_1}}{\pi \cdot (R_{max} + R_{min}) \cdot G} = \frac{\frac{F_1}{\cos \alpha_1}}{\pi \cdot d_2 \cdot \frac{H_1}{\cos \alpha_1}} = \frac{F_1}{\pi \cdot d_2 \cdot H_1} = \frac{F}{\pi \cdot z \cdot d_2 \cdot H_1} \leq q_a$$

To dimension the threaded assembly, a series of coefficients are introduced:

- Coefficient of thread height: $\psi_h = \frac{H_1}{p}$, (0.5 is adopted for trapezoidal profile and 0.75 for saw profile).
- Coefficient of the threaded length of the nut: $\psi_m = \frac{m}{d_2} = \frac{z \cdot p}{d_2}$ (is adopted between 1,2...2,5).

From the contact stress relation q , the required pitch diameter of external thread results.

$$d_{2nec} = \sqrt{\frac{F}{\pi \cdot \psi_m \cdot \psi_h \cdot q_a}}$$

Standardized thread elements are adopted based on the following condition:

$$d_{2STAS} \geq d_{2nec}$$

Thread bending

For the screw, it is considered that the thread coil is fixed on the cylinder of diameter d_1 (Figure 1.12).

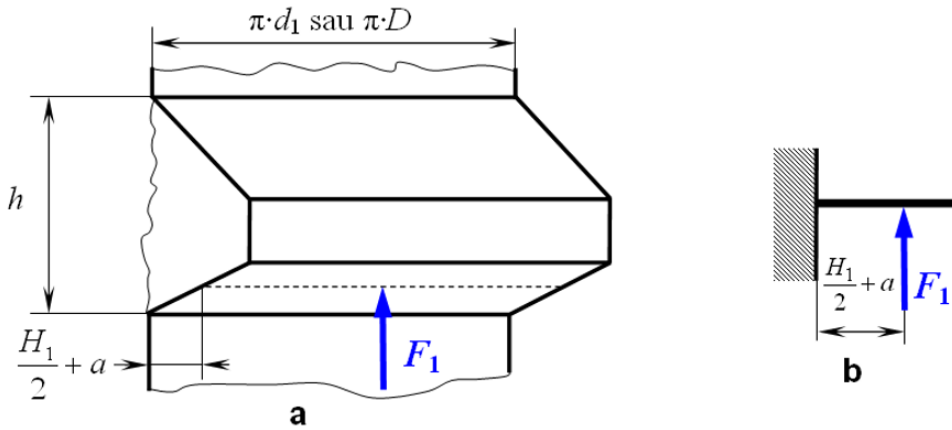


Fig. 1.12 Bending of the thread coil

$$\sigma_i = \frac{M_i}{W} = \frac{F_1 \cdot \left(\frac{H_1}{2} + a \right)}{\pi d_1 \cdot \frac{h^2}{6}} = \frac{6F \cdot \left(\frac{H_1}{2} + a \right)}{z \cdot \pi \cdot d_1 \cdot h^2} \leq \sigma_{ia}$$

The nut is checked using the relationship:

$$\sigma_i = \frac{6F \cdot \left(\frac{H_1}{2} + a \right)}{z \cdot \pi \cdot D \cdot h^2} \leq \sigma_{ia}$$

Thread Shearing

Shearing of the thread coils is a stress of minor importance, with the rupture section being the same as in the case of bending stress.

$$\tau_f = \frac{F_1}{\pi \cdot d_1 \cdot h} = \frac{F}{z \cdot \pi \cdot d_1 \cdot h} \leq \tau_{af}$$

For the nut:

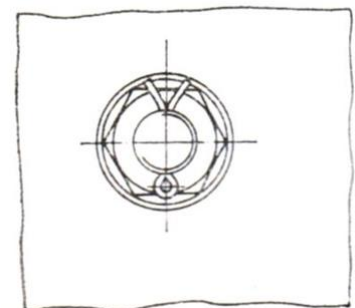
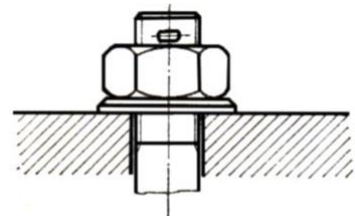
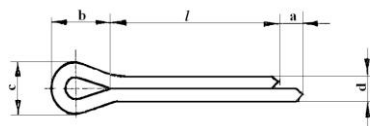
$$\tau_f = \frac{F}{z \cdot \pi \cdot D \cdot h} \leq \tau_{af}$$

Ensuring threaded assemblies

Methods of securing without introducing additional stresses into the assembly

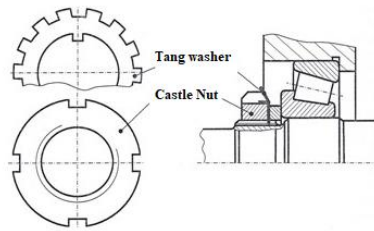
- „R-Clip or spindles”

.....



- Using various types of lock washers

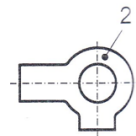
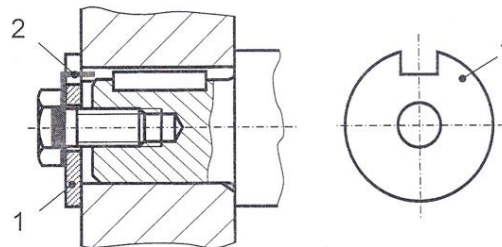
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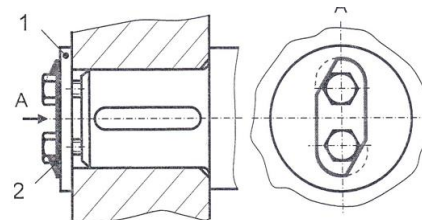
$d \leq 28$ mm (Using a locking screw)

- Securing parts on shaft ends

.....



$d > 28$ mm (Using two locking screws)



Methods of securing with the introduction of additional stresses into the assembly

- Securing with a counterlock nut

.....

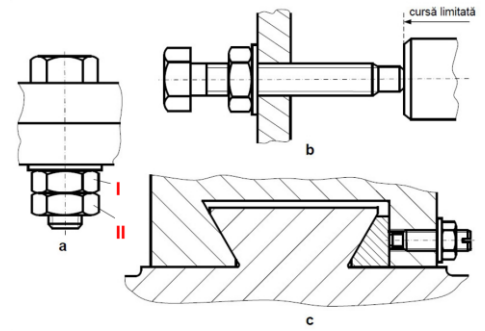
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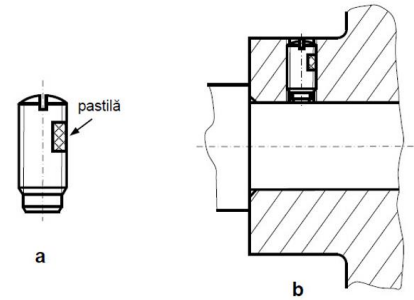


- Self-locking pins

.....

.....

.....



- Self-locking nuts

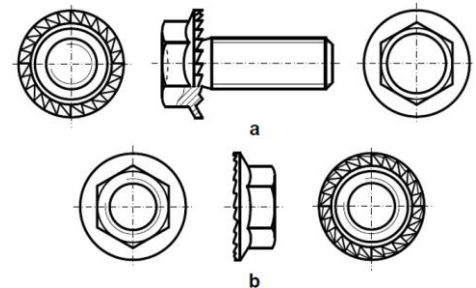
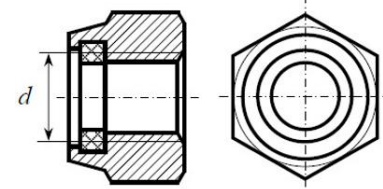
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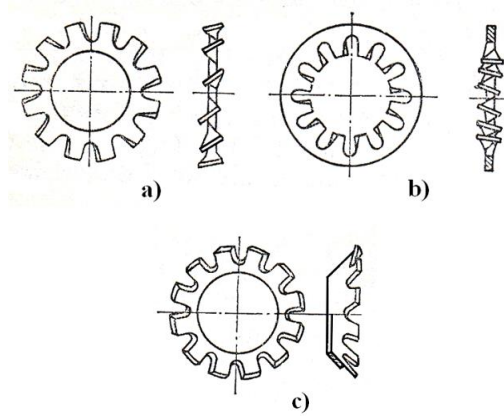
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- Tooth washers

.....



Additional notes / Observations

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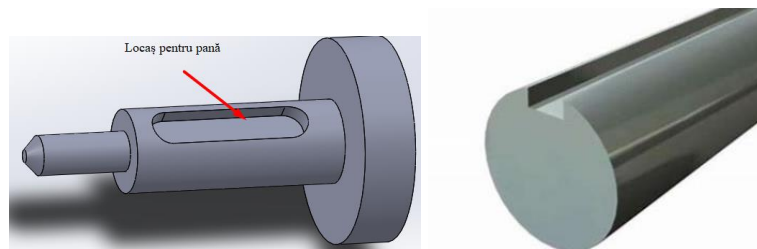
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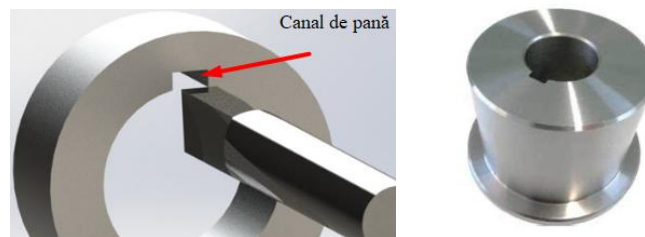
2. ASSEMBLIES WITH KEYS AND SPLINES

Key assemblies

These are detachable assemblies, formed by shape, and serve to transmit torques in the case of longitudinal keys or a force in the case of transverse keys. In order to enable the assembly and operation of the joint, a keyseat will be machined into the shaft (by milling), and a keyway will be created in the hub (by mortising or broaching). Figure 2.1 shows such examples.



(a) Examples of key seats machined into the shaft



(b) Examples of keyways created in the hub

Fig. 2.1 Keyseats and keyways

In Figure 2.2, examples of parallel keys used (or not used) in shaft-hub-wheel assemblies are presented. Figure 2.3 shows other types of keys, while Figure 2.4 illustrates an assembly example.

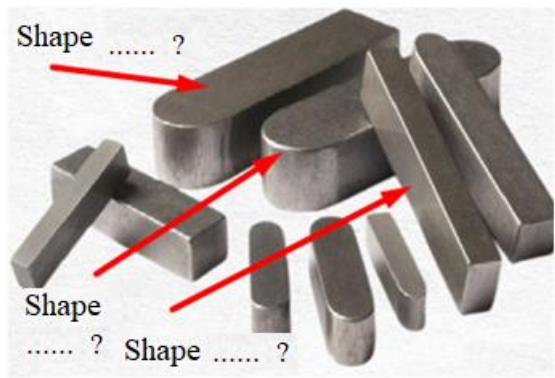


Fig. 2.2 Examples of parallel keys by shape

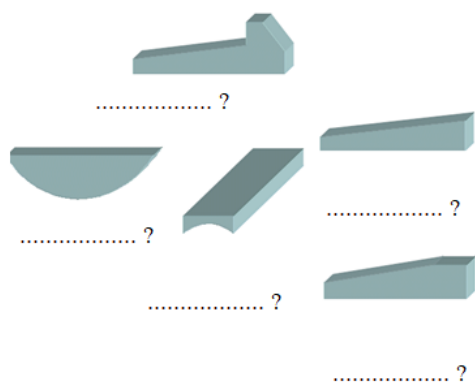


Fig. 2.3 Other types of keys.

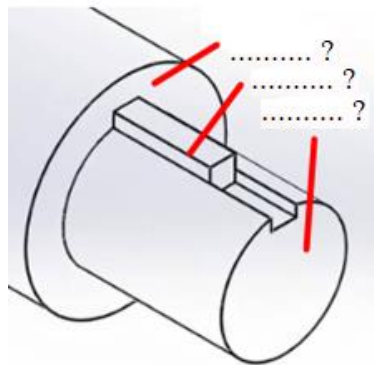


Fig. 2.4 Assembly with parallel key type B

Assemblies with keys can be classified according to the following criteria:

✓ **Based on the assembly method**

-
-

✓ **Based on the structural form**

-
-
-
-
-
.....

✓ **Based on the ratio of section dimensions**

-
-

✓ **Based on the shape of the ends**

-
 -
 -
-

•

•

For the analysis of stresses in key assemblies, consider figure 2.5. During the transmission of the torsion torque T from the shaft to the hub, the key is subjected to contact pressure/crushing stress in height and shear stress in width.

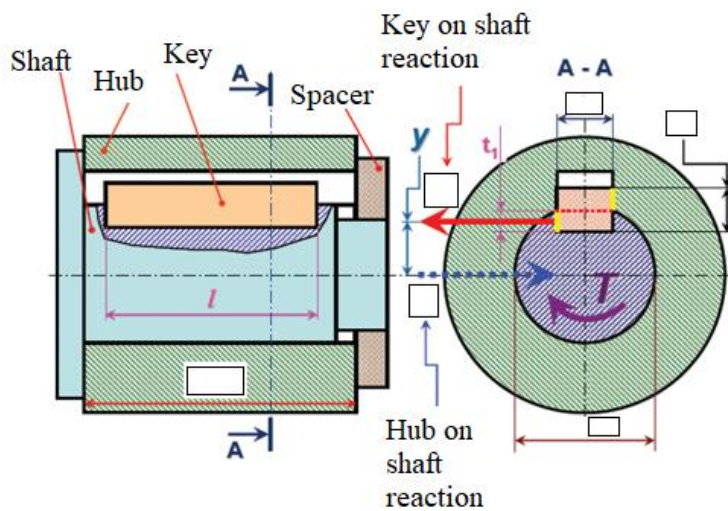


Fig. 2.5 Stresses in parallel key assemblies

.....

.....

.....

.....

.....

.....

 Dynamic equilibrium equation of the shaft:

$$T = F \times y \cong Fx \frac{d}{2}, \text{ therefore } F = -.$$

Parallel key is stressed at on the lateral faces and to..... in the contact surface between the shaft and the hub.

$$\sigma_s = \frac{2 \cdot F}{h \cdot l} = \frac{4 \cdot T}{h \cdot l \cdot d} \leq \sigma_{as} \text{ (.....)}$$

$$\tau_f = \frac{F}{b \cdot l} = \frac{2 \cdot T}{b \cdot l \cdot d} \leq \tau_{af} \text{ (.....)}.$$

$$l_{c\sigma} = \frac{4 \cdot T}{h \cdot d \cdot \sigma_{sa}}$$

(The necessary length of the key to prevent its crushing).

$$l_{cf} = \frac{2 \cdot T}{b \cdot d \cdot \tau_{fa}}$$

(the necessary length of the key in case shear verification is not performed.).

In both situations, according to *ISO 2491*, is chosen:

.....

Key marking:

Key **A b x h x l** –

Where: A –

b –

h –

l –

Materials and technology:

Steels with a minimum tensile strength of 600 MPa, typically according to SR EN 10025 as E 335 (OL 60), or according to SR EN 10083 as 1 C 45 (OLC 45).

Disc key assemblies

In figure 2.6, an example of a disc key assembly is presented.

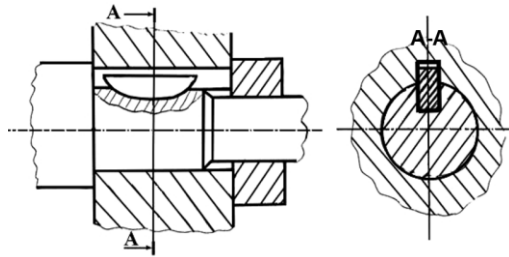


Fig. 2.6 Disc key assembly

A disc key is.....

.....

The lower part of the key (the semicircular disc) is mounted in.....

The groove in the hub is typically made. Once placed in the correct position, the disc key tilts and self-aligns on the shaft.

Advantages	Disadvantages
.....	

.....	
.....	
.....	
.....	
.....	

Other types of keys

Longitudinal tightening keys

They have an inclined upper face, therefore

.....

The torque transmission is achieved through friction.

Disadvantages:

.....

.....

.....

Calculating inclined keys

Hypotheses:

The torque transmitted through such a joint is given by the sum of the moments of all forces relative to the axis of the shaft:

1.

.....

.....

2.

In figure 2.7, the forces acting in an inclined key assembly are presented, necessary for determining the transmitted torque moment.

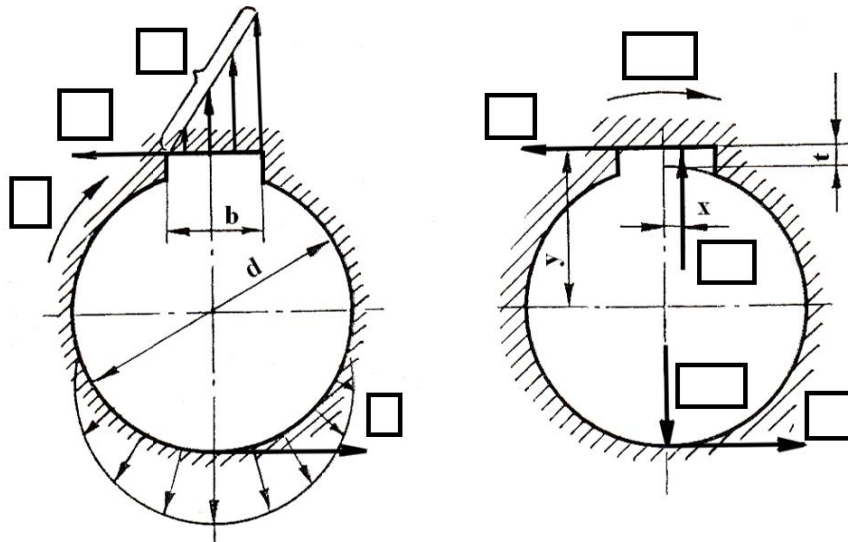


Fig. 2.7 Forces in an assembly with inclined keys

$$M_t = Q \cdot x + \mu \cdot Q \cdot y + \frac{d}{2} \cdot F_f, \text{ where:}$$

$Q =$

$p -$

$l -$

$x =$

$y \approx$

[illegible]

Spline assemblies

Spline assemblies are shaft-hub connections designed to transmit torque and rotational motion.

In the case of splined shafts (figure 2.8), the splines can be somewhat considered as "multiple longitudinal keys," forming a common body with the shaft. Although they are primarily used in spline couplings (allowing the mounting and axial movement of gear wheels in gearboxes), they are sometimes also used in fixed assemblies.

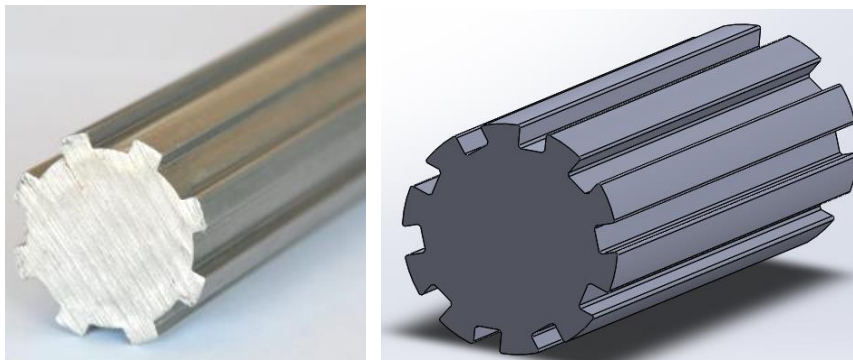


Fig. 2.8 Splined shaft

Advantages	Disadvantages
.....	
.....	
.....	
.....	

Splined hubs (figure 2.9) are made using technological processes such as slotting (less demanding execution) or broaching (higher precision, used for mass production).



Fig. 2.9 Splined hubs

Classification of splines based on the profile:

1.
2.
3.

In figure 2.10 there are several examples of splines based on their profil.

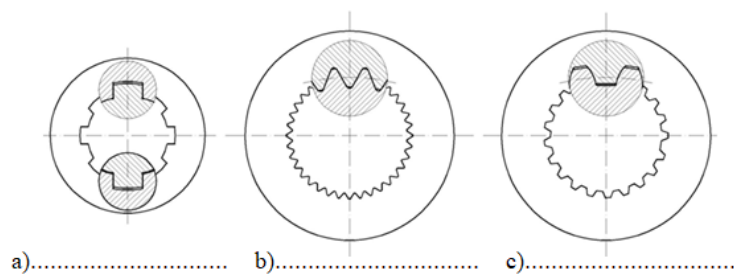


Fig. 2.10 Examples of splines classified by profile

Splines with rectangular profile are the most commonly used. In their case, standards specify:

-;
-;
-;
-;
-

Depending on the **centering method**, rectangular splines are divided into three categories (figure 2.11):

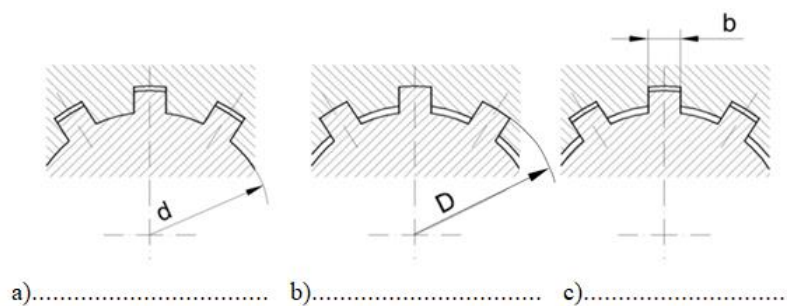


Fig. 2.11 Centration possibilities of rectangular splines

- a)

;
- b)

;
- c)

Depending on the **load capacity and coupling method**, standards divide rectangular splines into **three series of sizes**, distinguished by the height of the splines, their number, and the method of centering.

- a)

 ;
- b)

 ;
- c)

Calculation of splined assemblies

The torque is transmitted from the shaft to the hub through the contact on the lateral faces of the shaft's protrusions and the splined hub (as shown in figure 2.12).

The stresses in spline assemblies are:

- a) ;
 b) ;
 c)

Obs. The most important stress is:.....

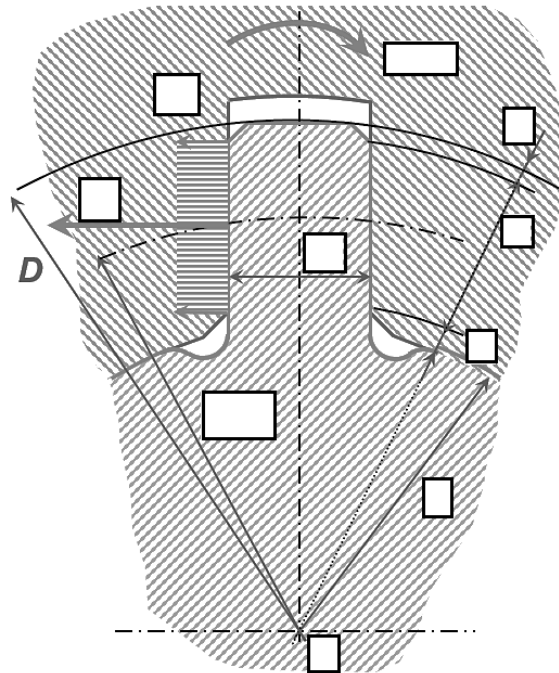


Fig. 2.12 Geometric elements in spline assemblies

The force on a spline:

$$F_1 = \sigma_s \cdot A = \sigma_s \cdot h \cdot L = \sigma_s \cdot \left(\frac{D-d}{2} - 2c \right) \cdot L \quad (\text{.....})$$

Dynamic equilibrium equation for the hub:

$$M_t = F_1 \frac{D_m}{2} \cdot k \cdot z \quad (\text{.....})$$

.....

Flanks crushing (at contact pressure):

$$\sigma_s = \frac{M_t}{k \cdot z \cdot L \left(\frac{D-d}{2} - 2c \right) \cdot \frac{D+d}{4}} \leq \sigma_{sa} \quad (\dots\dots\dots)$$

The necessary length of the assembly:

$$L_{\min} \geq \frac{2 \cdot M_t}{k \cdot D_m \cdot z \cdot \left(\frac{D-d}{2} - 2c \right) \cdot \sigma_{as}}, \text{ where } k = 0,75 \text{ este}$$

..... ;

D -

d -

c -

where σ_{as} the permissible crush strenght, provided in the standard according to the working conditions of the assembly (....., ..., ...) and also operation and coupling mode.

(....., ...).

Verification of the spline assembly involves:

$$L \geq L_{\min}, \text{ therefore:}$$

$$M_{t\,cap} = \frac{1}{2} \cdot k_{\sigma} \cdot d_m \cdot z \cdot L \cdot \sigma_{as} \cdot \left(\frac{D-d}{2} - 2c \right) \geq M_{tn},$$

where L is and $M_{t\,cap}$

.....

Design algorithm

The sizing of an assembly with rectangular grooves generally starts from knowing the nominal torque moment M_{tn} , the functional role of the assembly, and the working conditions. If the inner diameter of the splined shaft is not known, it is preliminarily sized based on the torsion strength condition:

$$d = \sqrt[3]{\frac{16 \cdot M_t}{\pi \cdot \tau_{ta}}}, \text{ where } \tau_{at} = 15 \dots 25 \text{ MPa}$$

is the admissible stress for torsion.

The size series of the splined assembly (light, medium, or heavy) is chosen based on the type of assembly and coupling conditions.

They are chosen, based on the **inner diameter** d from the preliminary sizing, from the standard of the selected series:

- d ;
- D ;
- b ;
- z ;
- C ;
-

The **necessary length** L_{nec} of the splines is determined.

If the length of the splined hub is known, a **verification calculation** is performed.

3. ASSEMBLIES WITH PINS AND BOLTS

Assemblies with pins

Pins are detachable assembly components used for the following purposes:

- ;
- ;
-

Pins can be classified according to the shape of their longitudinal section into:

- (SR EN ISO 2338/ SR EN ISO 2339),
- OR

Depending on the shape of the outer surface, pins can be divided as follows:

- OR

Below are presented several examples: solid cylindrical (Figure 3.1), with spherical head (Figure 3.2a, flat or straight), tubular cylindrical (Figure 3.2b), conical (Figure 3.3a with threaded plug, b with split end, c grooved), threaded pins (Figure 3.4a chamfered, b with plug, c with conical tip).

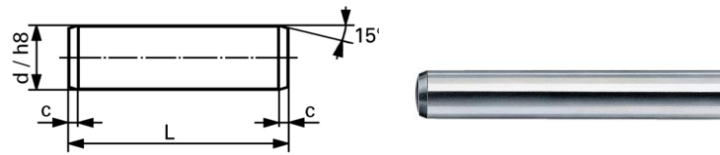
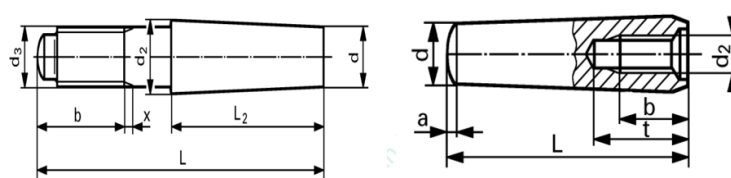


Fig. 3.1 Solid cylindrical pin

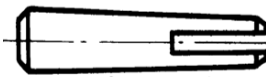


Fig 3.2 a) Spherical head pin

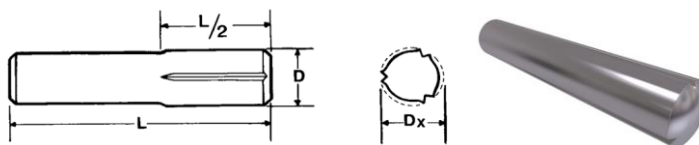
b) Cylindrical tubular pin



a. Conical pin with threaded plug



b. Pin with with split end



c. Grooved pin

Fig. 3.3 Conical pins

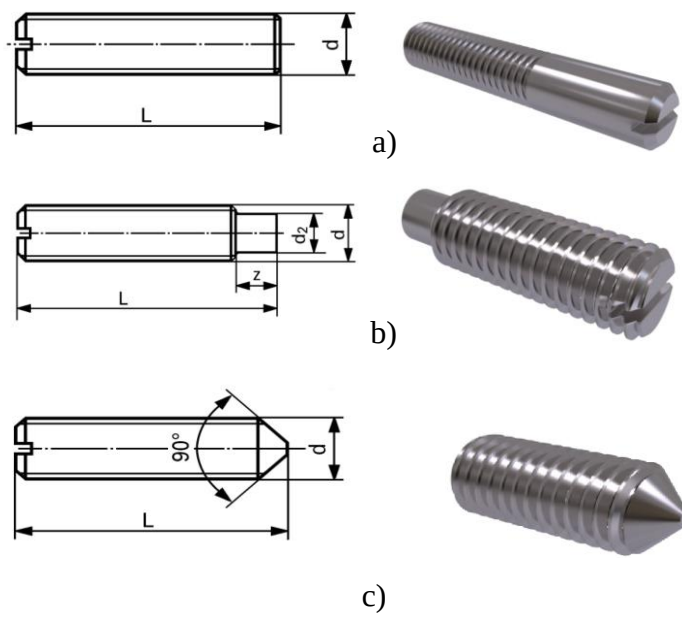


Fig. 3.4 Threaded pins a) Chamfered b) With plug c) With conical tip

Solid cylindrical pins:.....

.....

.....

Tubular cylindrical pins:.....

.....

.....

Conical pins:.....

.....

.....

.....

.....

Threaded pins:.....

.....

The calculation of pins

Locking pins are typically used in shaft-hub assemblies that transmit torsion torques through direct contact from the shaft to the pin, and then from the pin to the hub (or vice versa).

The stresses that occur in the assembly are and

The calculation is carried out under the assumption that the pin is fitted tightly into the hub and has clearance in the shaft (Figure 3.5).

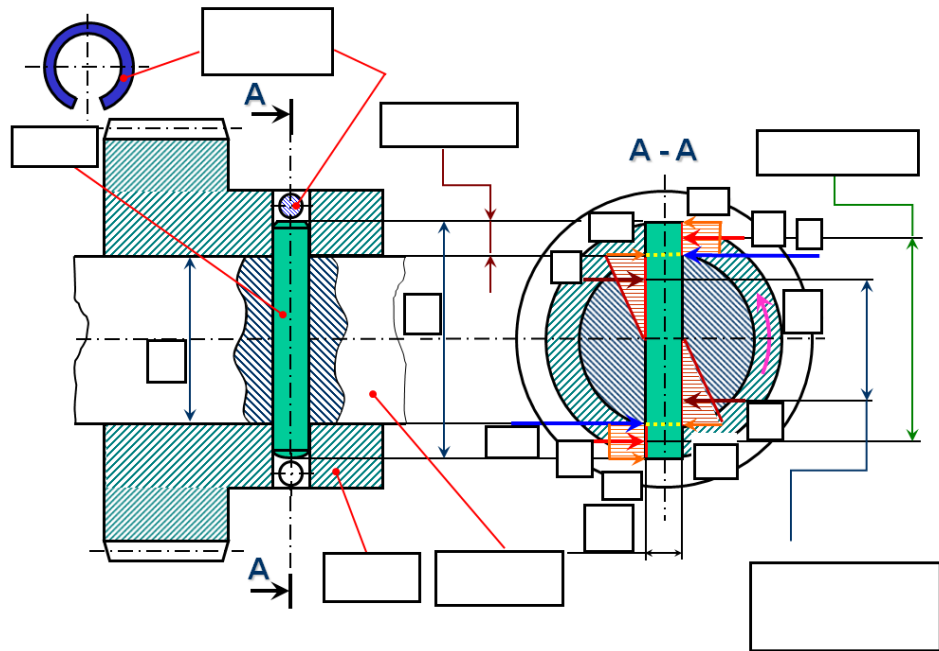


Fig. 3.5 Pin assembly (shaft-hub)

Shearing stress:

$$M_t = F \cdot d \Rightarrow \dots F = - \quad (....)$$

$$\tau_f = \frac{2 \cdot F}{\pi \cdot d_s^2} = \frac{2 \cdot M_t}{\pi \cdot d \cdot d_s^2} \leq \tau_{fa} \quad (.....)$$

The capable torque to transmit:

$$M_{t_max} = \frac{\pi}{2} \cdot d \cdot d_s^2 \cdot \tau_{fa} \quad (.....)$$

The required diameter of the pin:

$$d_{s_nec} = \sqrt{\frac{2 \cdot M_{t_max}}{\pi \cdot d \cdot \tau_{fa}}} \quad (.....)$$

Crushing/contact pressure stress:

Pin-hub assembly:

$$M_t = F_1 \cdot \frac{l+d}{2} \Rightarrow \dots F_1 = - \quad (.....)$$

$$\sigma_{s1} = \frac{2 \cdot F_1}{d_s \cdot (l-d)} = \frac{4 \cdot M_t}{d_s \cdot (l^2 - d^2)} \leq \sigma_{sa1} \quad (.....)$$

Pin-shaft assembly:

$$M_t = F_2 \cdot \frac{2 \cdot d}{3} = \sigma_{s2} \left(\frac{1}{2} d_s \cdot \frac{d}{2} \right) \frac{2d}{3} = \frac{1}{6} d^2 d_s \sigma_{s2} \quad (.....)$$

$$\sigma_{s2} = \frac{6 \cdot M_t}{d_s \cdot d^2} \leq \sigma_{sa2} \quad (.....),$$

where: M_t - ;

d_s - ;

d - ;

l – ;

τ_{af} -;

Obs.: $\sigma_{as} \leq 0,8 \cdot \sigma_{02}$, with the specification that in the case of grooved pins, the values are taken as 30% smaller than those used in the calculation of smooth pins;

$$\tau_{af} = (0,2 \dots 0,3) \cdot \sigma_{02}.$$

Assemblies with bolts

Bolts are cylindrical machine components used as connecting parts in joints. They are constructed in two basic forms:

- (ISO 2341),

namely (ISO 2338), executed in two variants (form A, B);

-,

also in two constructive variants.

All basic forms are executed in two variants (Figure 3.6 and 3.7):

- , form A (fig. a, b, e);
- , form B (fig. c, d și f).

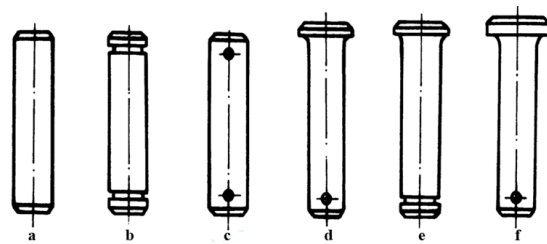


Fig. 3.6 Different types of bolts

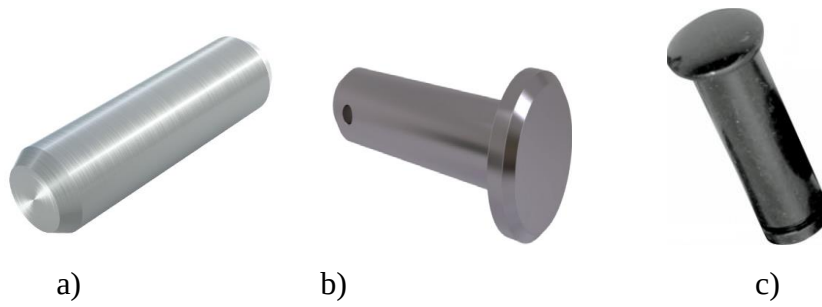


Fig. 3.7 a) Bolt without clip hole (form A), b) Bolt with clip hole (form B), c) Bolt with retaining groove

Bolts can also be equipped with grooves for retaining rings (Fig. b and e).

The calculation of bolts

For the calculation of bolts, the assembly shown in Figure 3.8 is proposed.

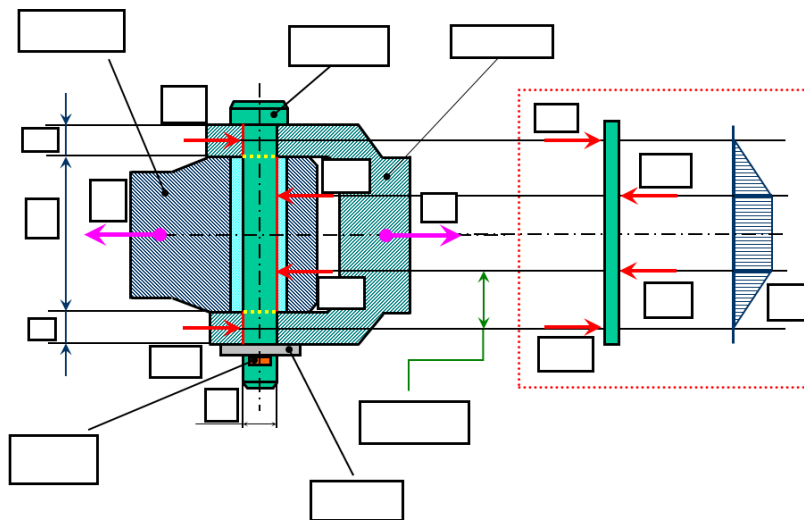


Fig. 3.8 Assembly with bolt

Crushing/contact pressure stress:

Clevis-bolt:

$$\sigma_{s1} = \frac{F}{2 \cdot s \cdot d_b} \leq \sigma_{s1a} \quad (\dots\dots\dots)$$

Tie rod-bolt:

$$\sigma_{s2} = \frac{F}{h \cdot d_b} \leq \sigma_{s2a} \quad (\dots\dots\dots)$$

Shear stress (bolt)

$$\tau_f = \frac{2 \cdot F}{\pi \cdot d_b^2} \leq \tau_{fa} \quad (\dots\dots\dots)$$

Bending moment:

$$M_i = \frac{F}{2} \cdot \left(\frac{s}{2} + \frac{h}{4} \right) = \frac{F \cdot (2 \cdot s + h)}{8} \quad (\dots\dots\dots)$$

Bending stress

$$\sigma_i = \frac{M_i}{W} = \frac{F \cdot (2 \cdot s + h)}{8 \cdot \frac{\pi \cdot d_b^3}{32}} = \frac{4 \cdot F \cdot (2 \cdot s + h)}{\pi \cdot d_b^3} \leq \sigma_{ia} \quad (\dots\dots\dots)$$

[illegible]

4. ASSEMBLIES WITH ELASTIC CLAMPS AND WITH SELF-PRESS FIT

Assemblies with elastic clamps (bracelets)

The assembly is done by tightening an encompassing elastic piece onto a shaft, using friction forces that arise from the more or less uniform pressing of the inner contact surface of the encompassing piece (clamp) onto the outer surface of the encompassed piece (shaft). This type of assembly is capable of transmitting small torque moments and ensuring the positioning of parts mounted on shafts, supports, pipes, etc.

Tightening represents:.....

..... .

The advantages of this type of assembly:.....

..... .

The materials used:.....

..... .

The calculation of assemblies with elastic clamps (bracelets)

For the development of calculation hypotheses, these assemblies can be found in two variants (as shown in Figures 4.1 and 4.2):

1.

2.

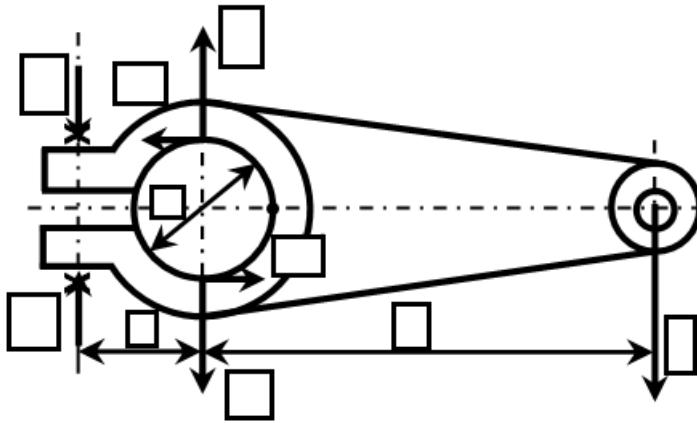


Fig. 4.1 Clamp with a split ring

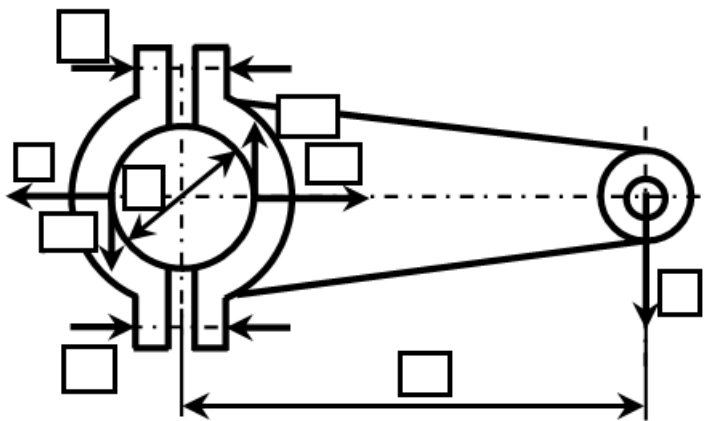


Fig. 4.2 Clamp with two rings

In figures 4.3 a) and b), two examples of using clamps (bracelets) are presented: a) with a split ring, for positioning and supporting the motorcycle handlebar; b) with two rings, for supporting and taking torsional moments from the stabilizer bar of a truck.



a) With a split ring



b) With two rings

Fig. 4.3 Examples of clamps

The following calculation hypotheses are established:

Force F_N :

A :

It is not taken into account:

The moment equation is written around point A, as:

$$i \cdot F_s \cdot \left(a + \frac{d}{2} \right) = F_N \cdot \frac{d}{2}$$

The sum of moments about point O is:

$$F \cdot l = \mu \cdot F_N \cdot d \Rightarrow F_N = \frac{F \cdot l}{\mu \cdot d}$$

By substituting into the above relationship, the tightening force is obtained.

$$i \cdot F_s \cdot \left(a + \frac{d}{2} \right) = \frac{F \cdot l}{\mu \cdot d} \cdot \frac{d}{2}$$

$$F_s = \frac{F \cdot l}{2\mu \cdot i \left(a + \frac{d}{2} \right)}$$

where:

F_N —....., [N];

F_s —....., [N];

μ —

d —, [mm];

a —, [mm].

l —, [mm].

i —

In the case of a clamp/bracelet assembled with initial clearance, the capable torsional moment can be calculated as follows:

$$T_{cap} = \frac{4}{\pi} \cdot 10^{-3} \cdot F_s \cdot \mu \cdot (2 \cdot a + d) \text{ [N}\cdot\text{m]},$$

Press fit (interference fit) assemblies

This type of assembly is achieved through tightening caused by the elastic deformation of the joined parts, without intermediate elements. Thus, frictional forces oppose the disassembly.

Classification:

—

[illegible]

Calculation elements:

- a) Determining the tightening according to the minimum pressure required to handle trust force or torsional moment;
- b) Knowing the uniform stresses generated in parts due to tightening effects
- c) Verifying the contact pressure;
- d) Calculating the deformations resulting from tightening (affects the proper functioning of the assembly).

To identify the necessary calculation elements, in Figure 4.4 there is a presentation of a self-tightening assembly (press fit), a) before pressing and b) after pressing, and c) a cross-section of the assembly.

Calculation hypothesis:

All calculations are performed in the elastic range of deformation.

After deformation, the same diameter d is obtained for the shaft and hub, because:

- d_a (.....) it decreases by Δa .
- d_b (.....) it increases by Δb .
- $d_b < d < d_a$.

Thus, the theoretical tightening can be determined:

$$S = d_a - d_b = d + \Delta_a - (d - \Delta_b) = \Delta_a + \Delta_b \quad [\mu\text{m}];$$

$$\text{where } p = \frac{S \cdot 10^{-3}}{d \cdot \left(\frac{K_a}{E_a} + \frac{K_b}{E_b} \right)} \quad [\text{MPa}],$$

p: [MPa];

S: [μm];

d: [mm];

E_a, E_b : [MPa];

.....

K_a, K_b :

.....

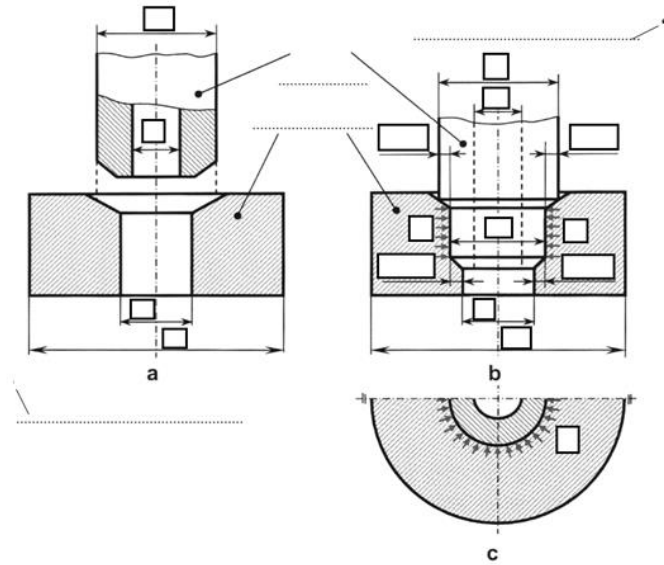


Fig. 4.4 Press fit assembly

The dimensionless coefficients that account for the radial deformations of the parts are determined as follows:

$$K_a = \frac{d^2 + d_1^2}{d^2 - d_1^2} - \nu_a$$

$$K_b = \frac{d_2^2 + d^2}{d_2^2 - d^2} + \nu_b, \text{ where:}$$

ν_a, ν_b :

d_1 :

d_2 :

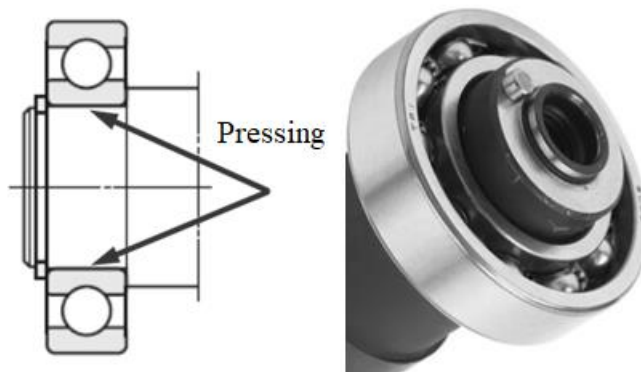
From the relationship of the contact pressure on the tightening surfaces, the theoretical tightening can be determined as follows:

$$S = p \cdot d \cdot \left(\frac{K_a}{E_a} + \frac{K_b}{E_b} \right) \cdot 10^3 \text{ } [\mu\text{m}].$$

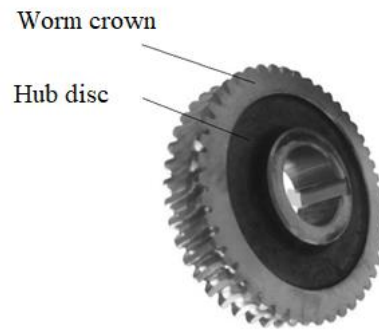
From the perspective of contact pressures indirectly related to tightening, there can be two extreme situations that should be avoided:

1. ;
2.

In figure 4.5, two press/interference fit assemblies are exemplified:
 a) mounting a bearing on a shaft and b) mounting a toothed crown on the hub disc of a worm wheel (for transmitting torsional moments).



a) Bearing-shaft assembly



b) Toothed worm crown on hub disc

Fig. 4.5 Examples of using press fit assemblies

In reality, the theoretical tightening does not correspond to the actual tightening between the assembled parts ($S \neq S_e$), for reasons such as:

1. ;
2. ;
3. ;

The effective tightness can be calculated:

$$S_e = S - (C_r \pm C_t + C_n) \quad [\mu\text{m}], \text{ where:}$$

C_r :
.....;

$$C_r = 5,5 \cdot (R_a^a + R_a^b) = 1,2 \cdot (R_z^a + R_z^b) \quad [\mu\text{m}], \text{ where:}$$

R_a^a ; R_a^b :
..... $[\mu\text{m}]$;

R_z^a ; R_z^b :
..... $[\mu\text{m}]$.

C_t :
;

$$C_t = (\alpha_b - \alpha_a) \cdot d \cdot (t - t_0) \cdot 10^3 \text{ } [\mu\text{m}], \text{ where:}$$

α_a, α_b :;

t_0 :;

t :

C_n :

[illegible]

5. SPRINGS

The spring is a machine part that, due to its shape and elastic material, transforms mechanical work into potential energy through its elastic deformation and is ability to reconvert (completely or partially) the accumulated potential energy back into mechanical work..

It is used for :

- ;
- ;
- ;
- ;
-

Classification of springs:

According to their construction form: (eg. fig. 5.1):

- ;
- ;
- ;
- ;
- ;
- ;
-

According to the direction and sense of the external force:

- ;
- ;
- ;

-
- ;
 -

According to the shape of the semifinished product's cross-section:

- ;
- ;
- ;
-

According to their functional role:

- ;
- ;
- ;
- ;
- ;
-

According to the material:

- ;
- ;
-

According to stiffness:

- ;
-

Materials used in the construction of springs:

1. ;
 2. ;
 3.
-

Requirements imposed on materials for springs:

- ;
- ;
- ;
- ;
- ;
- ;
- ;



Fig. 5.1 Various types of springs: a. Leaf springs, b. Compression springs, c. Tensile springs, d. Variable pitch springs, e. Disc springs, f. Ring springs, g. Spiral spring, h. Circular sealing springs, i. Torsion bar springs, j. Torsion springs, k. Concave spring.

The **characteristic of a spring** is the curve that represents the relationship between the load acting on the spring (force, moment) and the elastic deformation it produces (deflection, angle). The graphical representation of the stiffness of a spring (its characteristic) is illustrated in Figure 5.2.

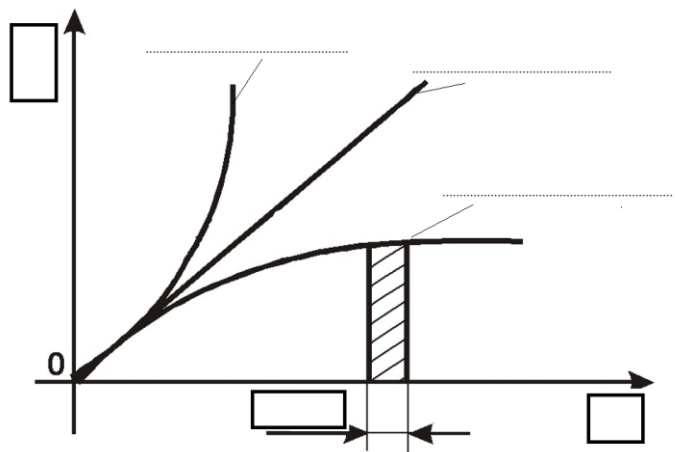


Fig. 5.2 Caracteristica arcului

The characteristic of the spring:

- ;
- ;
-

Linear characteristic:

- ;
- ;
-

Clarification: With these springs, there is proportionality between forces and deformations (Fig. 5.2):

$$\frac{F_i}{f_i} = \tan \alpha = C, \text{ where:}$$

C : ;

F_i : ;

f_i :

The elementary mechanical work stored is (Fig. 5.2):

$$dL = F \cdot df$$

The area under the spring characteristic curve and the abscissa represents the total mechanical work stored by the deformation of the spring:

$$L = \frac{1}{2} F \cdot f = \frac{1}{2} C \cdot f^2$$

Analogous to springs subjected to moments:

$$L = \frac{1}{2} M_t \cdot \theta = \frac{1}{2} C \cdot \theta^2$$

In the case of springs with variable stiffness:

$$L = \int_0^{f_n} F \cdot df; \quad L = \int_0^{\theta_n} M_t \cdot d\theta$$

Observation:

.....

Calculation of the main types of springs used in industry

Leaf springs (bending load). The single leaf of these springs, typically fixed at one end and under the action of external load F at the free end, can have a rectangular, triangular, trapezoidal, etc., cross-section projection. The height of the leaf can be constant or variable.

The unit stress of the leaf spring (rectangular), at a section x distance from the point of fixation, is (Figure 5.3):

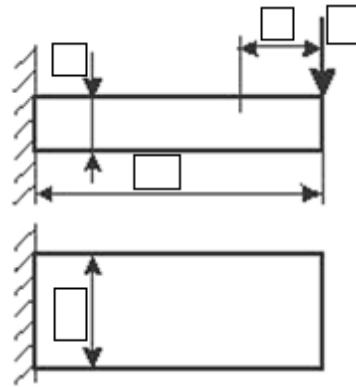


Fig. 5.3 Rectangular leaf spring

$$\sigma_{ix} = \frac{F \cdot x}{W} = \frac{6 \cdot F \cdot x}{b \cdot h^2}$$

$$\sigma_{i0} = 0 \text{ for } x = 0 \text{ iar } \sigma_{imax} = \frac{6 \cdot F \cdot l}{b \cdot h^2} \text{ for } x = l.$$

The maximum force supported by the spring:

$$F_{\max} = \frac{bh^2}{6} \cdot \frac{\sigma_{imax}}{l}$$

Free end deflection:

$$f_{\max} = \frac{F \cdot l^3}{3 \cdot E \cdot I} = \frac{b \cdot h^2 \cdot l^2}{18 \cdot E \cdot I} \cdot \sigma_{\max} = \frac{12 \cdot b \cdot h^2 \cdot l^2}{18 \cdot E \cdot b \cdot h^3} \cdot \sigma_{\max} = \frac{2}{3} \frac{\sigma_{\max}}{E} \frac{l^2}{h}; \quad \text{where } I = \frac{b \cdot h^3}{12}$$

Stored mechanical work:

$$L = \frac{1}{2} \cdot F \cdot f_{\max} = \frac{1}{2} \cdot \frac{b \cdot h^2}{6 \cdot l} \sigma_{\max} \cdot \frac{2}{3} \frac{l^2}{E \cdot h} \sigma_{\max} = \frac{1}{18} \cdot V \cdot \frac{\sigma_{\max}^2}{E}$$

Torsion bar spring

Torsion bar springs are formed from straight bars subjected to torsion, with moments applied at the ends (Figure 5.4).

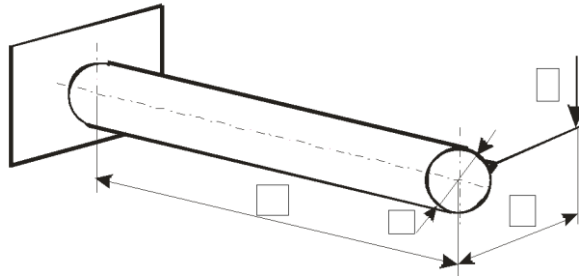


Fig. 5.4 Torsion bar spring (circular section)

$$T = F \cdot R = W_p \cdot \tau_t = \frac{\pi \cdot d^3}{16} \tau_t$$

$$d = \sqrt[3]{\frac{16 \cdot T}{\pi \cdot \tau_{ta}}}$$

$$\tau_t = \frac{16 \cdot T}{\pi \cdot d^3} \leq \tau_{ta}$$

Angular deformation:

$$\theta = \frac{T \cdot l}{G \cdot I_p} = \frac{32 \cdot F \cdot R \cdot l}{G \cdot \pi \cdot d^4} = 2 \cdot \frac{\tau_t}{G} \cdot \frac{l}{d}; \quad \text{where } I_p = \frac{\pi \cdot d^4}{32}$$

Stored mechanical work:

$$L = \frac{1}{2} \cdot T \cdot \theta = \frac{1}{2} \cdot \frac{\pi \cdot d^3}{16} \cdot \tau_{\max} \cdot \frac{2 \cdot \tau_{\max}}{G} \cdot \frac{1}{d}$$

$$L = \frac{1}{4} \cdot \frac{\tau_{\max}^2}{G} \cdot V = K \cdot \frac{\tau_{\max}^2}{G} \cdot V; \quad \text{where} \quad V = \frac{\pi \cdot d^2}{4} \cdot l$$

Helical springs

Helical springs are springs obtained by winding wire of any cross-section (circular, rectangular) in a helix pattern, on a guiding surface (Figure 5.5).

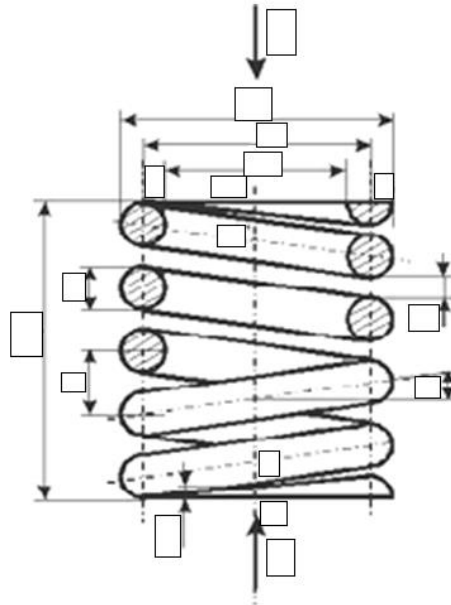


Fig. 5.5 Helical compression spring

Tension in the wire:

$$\tau_t = \frac{T}{W_p} = \frac{F \cdot R_m}{\frac{\pi \cdot d^3}{16}} = \frac{8 \cdot F \cdot D_m}{\pi \cdot d^3}$$

Unit stress:

$\tau_{\max} = k \cdot \tau_t$, where k is a shape factor coefficient.

If noted with $i = \frac{D_m}{d}$ şi $k = 1 + \frac{1.6}{i}$ (i - reeling ratio),

then $\tau_{\max} = k \cdot \frac{8 \cdot F \cdot D_m}{\pi \cdot d^3} = k \cdot \frac{8 \cdot F}{\pi \cdot d^2} \cdot i \leq \tau_{ta}$

Sizing is done using the relationship:

$$d = \sqrt[3]{k \cdot \frac{8 \cdot D_m \cdot F}{\pi \cdot \tau_{ta}}} = \sqrt[3]{k \cdot \frac{8 \cdot i \cdot F}{\pi \cdot \tau_{ta}}}$$

Deflection of the spring:

$$f = R_m \cdot \theta = \frac{D_m}{2} \cdot \frac{T \cdot l}{G \cdot I_p} = \frac{D_m}{2} \cdot \frac{F \cdot \frac{D_m}{2} \cdot \pi \cdot D_m \cdot n}{G \cdot \frac{\pi \cdot d^4}{32}}$$

$$f = \frac{8 \cdot D_m^3 \cdot n}{G \cdot d^4} \cdot F \quad (n - \text{active turns number}).$$

Mechanical work :

$$L = \frac{1}{2} \cdot F \cdot f = \frac{1}{2} \cdot \frac{8 \cdot D_m^3 \cdot n}{G \cdot d^4} \cdot F^2 = \frac{1}{2} \cdot \frac{8 \cdot D_m^3 \cdot n}{G \cdot d^4} \cdot \left(\tau_{\max} \cdot \frac{\pi \cdot d^3}{16} \cdot \frac{2}{D_m} \right)^2$$

$$L = \frac{1}{4} \cdot \frac{V}{G} \cdot \tau_{\max}^2 = K \cdot \frac{V}{G} \cdot \tau_{\max}^2, \text{ where : } V - \text{ is the volume of the spring, and}$$

k - is the volumetric utilization coefficient.

[illegible]

Introduction. Materials. Geometric elements

The gears are:.....
.....

Gearing represents:

.....

.....

[illegible]

Materials for gearings

The main materials are:.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Transmission ratio

If a gear has a very large number of teeth z , which tends to infinity, it turns into a gear rack. The gearing consisting of a gear and a rack serves to transform the rotational movement into translational movement and vice versa. Transmission ratio i_{12} in case of gearing is the ratio between..... ω_1 specific to the driving gear 1
..... ω_2 specific to the driven gear 2:

$$i_{12} = \frac{\omega_1}{\omega_2} = \frac{n_1}{n_2}$$

Gearing ratio u_{12} is the ratio between
..... z_2 and

z_1 :

$$u_{12} = \frac{z_2}{z_1}$$

It can be shown that the transmission ratio and the gearing ratio have the same value:

$$i_{12} = u_{12} .$$

The fundamental law of gearing

The law of gearing, known as Willis' theorem, establishes the condition that must be met by the profile curves bordering two teeth in contact so that the motion can be transmitted with a constant transmission ratio.

For two gears to transmit the rotational movement with a constant transmission ratio, it is necessary that:.....

.....

.....

.....

The point noted "C" (figure 6.1) is named:.....

.....

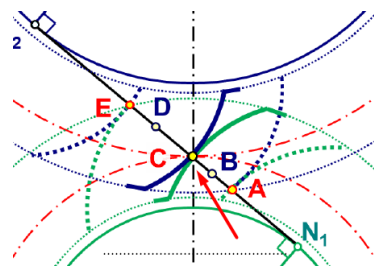


Fig. 6.1 The pitch point

The reference rack

If the number of teeth z tends to infinity, the gear wheel becomes a rack, the circles become straight, and the involute becomes a rectilinear profile.

The standardized geometric elements are defined on the reference rack.

Figure 6.2 shows a reference rack that contains the specific geometric elements.

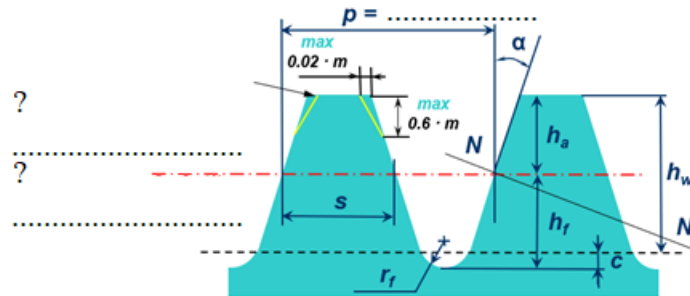


Fig. 6.2 Reference rack

where: $h_a^* = h_a / m = 1$

$\rho_f^* = \rho_f / m = 0,38$

$c^* = c / m = 0,25$

$\alpha = 20^\circ$

$m = \frac{p}{\pi}$

Teeth profile shifting

Two involute gears will mesh with each other if they each mesh with the same rack. Thus, the rack tool moves with respect to the reference line by the distance x (figure 6.3), which is expressed as a function of the module m :

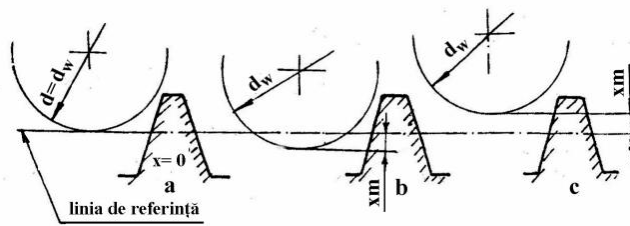


Fig. 6.3 Teeth profile shifting

If: $x > 0 \Rightarrow$ profile shifted gearwheel
(the rack moves closer to the center of the wheel relative to the reference position),

$x < 0 \Rightarrow$ profile shifted gearwheel
(the rack moves away from the center of the wheel relative to the reference position),

$x = 0 \Rightarrow$ gearwheels.

Covering degree

In order for a gear to transmit a continuously movement, with a constant transmission ratio, it is necessary that before a pair of teeth exits the gear, the next pair of teeth must already be engaged in meshing. The covering degree: $\varepsilon > 1$. At the beginning and end of the engagement of a pair of teeth, there is another pair of teeth in gear meshing, a stage called, in the rest of the time there is only one pair of teeth in gear meshing, a stage called..... (figure 6.1 and 6.4).

Cylindrical Spur Gears Calculation

The forces in Cylindrical Spur Gears

In gearing, the contact of the teeth under pressure moves from the tip of the tooth to its base. The point of application of the resulting contact pressure F_n , having the normal direction to the involute profile moves on the active flank continuously overlapping the common normal N-N.

As a result of sliding between the flanks, frictional forces arise which, added together with F_n , give a resultant that closes with the direction of the N-N normal an angle equal to the friction angle (figure 6.5).

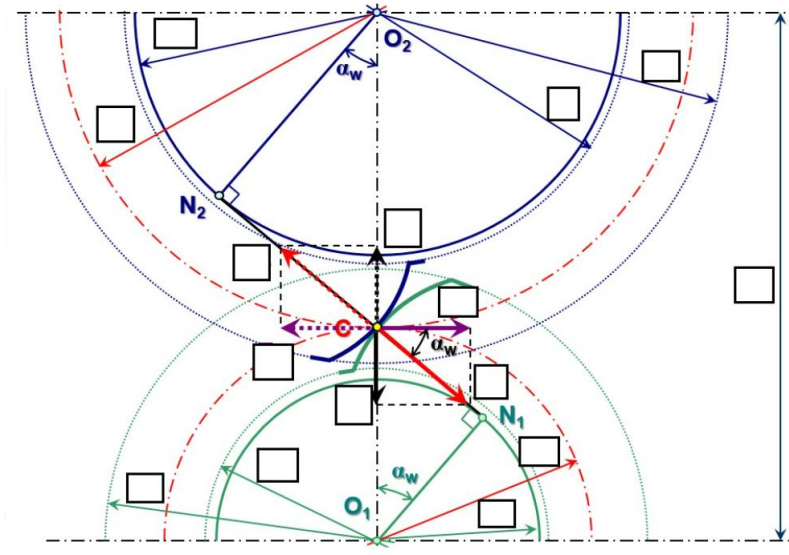


Fig. 6.5 Forces in Spur Gearing

Tangential force at the operating pitch circle:

Knowing that: $M_{t1}(T_1) = F_{t1} \cdot \frac{d_{w1}}{2}$, then $F_{t1} = \frac{2 \cdot M_{t1}}{d_{w1}} = F_{t2}$, and

$$M_{t1}(T_1) = 9550 \frac{P_1[\text{kW}]}{n_1[\text{rot/min}]} \quad [\text{Nm}].$$

The normal force at the operating pitch circle:

$$F_{n1} = \frac{F_{t1}}{\cos \alpha_w} = \frac{2 \times M_{t1}(T_1)}{d_{w1} \cos \alpha_w} = F_{n2}$$

The radial force at the operating pitch circle:

$$F_{r1} = F_{t1} \tan \alpha_w = F_{r2}, \text{ where:}$$

$M_{t1}(T_1)$;

d_{w1} ;

P_1, n_1 ;

α_w ;

$F_{t1, 2}$;

$F_{n1, 2}$;

$F_{r1, 2}$;

Factors influencing the forces in the gear

In reality, due to overloads, the forces that develop in the gear are influenced by factors such as:

The operating mode factor K_A

It highlights.....

, and the values
 of this factor depend.....

The dynamic factor K_v

This factor takes into account.....

$$\frac{v_w \cdot Z_1}{100} > 1, \text{ where } v_w \dots\dots\dots$$

The load distribution factor on the tooth width $K_{H\beta}$

This factor takes into account.....

Frontal plane load distribution factor $K_{H\alpha}$

It highlights.....

The load distribution factor on the tooth width $K_{F\beta}$ for bending stress

$$K_{F\beta} = K_{H\beta}^N, \text{ where } N = f\left(\frac{h}{b}\right), \text{ and } b \dots\dots\dots,$$

h.....

Frontal plane load distribution factor $K_{F\alpha}$ for bending stress

$$K_{F\alpha} = 1 + 2 \cdot (K_{H\alpha} - 1)$$

Strength calculation of cylindrical spur gears

Contact pressure calculation

To avoid pitting wear, the stresses σ_H which appear not to exceed the admissible fatigue contact stresses of the tooth flanks (σ_{HP}).

The linear contact between the flanks of two teeth is assimilated to the contact of two cylinders with radius equal to the radius of curvature of the teeth involutes in that contact point, the width equal to the teeth width **b** and load with the tooth force **F_n**.

Hertz's theory. It establishes the relationship for calculating the contact pressure at the contact of two elastic solid bodies (cylinders), figure 6.6.

Simplifying assumptions:

1. ;

2. ;
3. ;
4. ;
..... ;
5. ;
..... ;
6. ;
7.

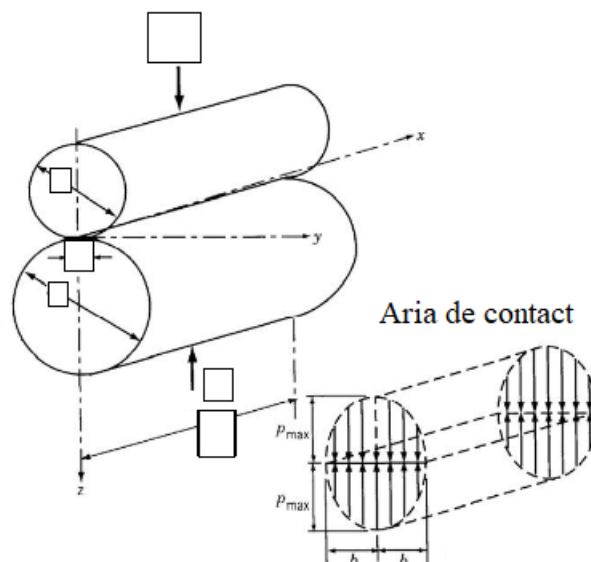


Fig. 6.6 Hertzian contact

The maximum contact pressure:

$$\sigma_H = \sqrt{\frac{F_n}{L_k} \cdot \frac{1}{\rho} \cdot \frac{1}{\pi \cdot \left(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right)}}, \text{ where } \rho - \text{reduced curvature radius}$$

$$\frac{1}{\rho} = \frac{1}{\rho_1} + \frac{1}{\rho_2}, \quad \text{and} \quad \begin{array}{l} v - \text{Poisson coefficient,} \\ E - \text{tensile modulus.} \end{array}$$

To avoid pitting wear that occurs due to tooth contact stresses, the σ_H stresses must not to exceed the admissible fatigue contact stresses of the tooth flanks (σ_{HP}).

The normal force:

$$F_n = F_{n1_ef} = \frac{2 \cdot T_1 \cdot C_H}{d_{w1} \cos \alpha_w}$$

Contact length:

$$L_k = \frac{b}{\frac{4 - \varepsilon_\alpha}{3}}$$

Reduced curvature radius:

$$\rho_1 = \frac{d_{w1}}{2} \cdot \sin \alpha_w, \quad \text{so}$$

$$\rho_2 = \frac{d_{w2}}{2} \cdot \sin \alpha_w$$

$$\frac{1}{\rho} = \frac{2}{d_{w1} \cdot \sin \alpha_w} \cdot \left(1 + \frac{1}{u} \right).$$

So, for the determination of contact stresses the expression becomes:

$$\sigma_H = \sqrt{\frac{1}{\pi \cdot \left(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right)}} \cdot \sqrt{\frac{2}{\sin \alpha_w \cos \alpha_w}} \cdot \sqrt{\frac{4 - \varepsilon_\alpha}{3}} \cdot \sqrt{\frac{2 \cdot T_1 \cdot C_H}{b \cdot d_{w1}^2} \cdot \frac{u+1}{u}}, \quad \text{but}$$

knowing that:

The elasticity factor of gear materials:

$$Z_E = \sqrt{\frac{1}{\pi \cdot \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)}}$$

The contact area factor:

$$Z_H = \sqrt{\frac{1}{\sin \alpha_w \cos \alpha_w}}$$

Coverage degree factor:

$$Z_\varepsilon = \sqrt{\frac{4-\varepsilon_\alpha}{3}}$$

It results in the gear check relationship on the Contact Pressure stress:

$$\sigma_H = Z_E \cdot Z_H \cdot Z_\varepsilon \cdot \sqrt{\frac{2 \cdot T_1 \cdot C_H}{b \cdot d_{w1}^2} \cdot \frac{u+1}{u}} \leq \sigma_{HP}, \text{ where:}$$

σ_{HP} —

Dimensioning of spur gear at the contact pressure stress

The gear width coefficient is taken into account: $b = \psi_a \cdot a_w$, where this one ψ_a is adopted depending on the ratio between power and shaft speed.

If the center distance a_w is written as:

$$a_w = \frac{d_{w1} + d_{w2}}{2} = \frac{d_{w1}(1+u)}{2},$$

and the pinion operating pitch circle diameter is:

$$d_{w1} = \frac{2 \cdot a_w}{1+u},$$

the Hertzian stress (Contact pressure) can be written:

$$\sigma_{HP} = Z_E \cdot Z_H \cdot Z_\varepsilon \cdot \sqrt{\frac{2 \cdot T_1 \cdot C_H}{\psi_a \cdot a_w \cdot \frac{4 \cdot a_w^2}{(1+u)^2}} \cdot \frac{u+1}{u}} = Z_E \cdot Z_H \cdot Z_\varepsilon \cdot \sqrt{\frac{T_1 \cdot C_H \cdot (1+u)^3}{2 \cdot \psi_a \cdot a_w^3 \cdot u}},$$

$$\sigma_{HP}^2 = Z_E^2 \cdot Z_H^2 \cdot Z_\varepsilon^2 \cdot \frac{T_1 \cdot C_H \cdot (1+u)^3}{2 \cdot \psi_a \cdot a_w^3 \cdot u},$$

In this way, the necessary center distance results, based on the contact pressure stress:

$$a_{w_nec} = (1+u) \cdot \sqrt[3]{\frac{T_1 \cdot C_H \cdot Z_E^2 \cdot Z_H^2 \cdot Z_\varepsilon^2}{2 \cdot \psi_a \cdot u \cdot \sigma_{HP}^2}}.$$

Calculation of cylindrical spur gear at bending stress

Simplifying assumptions:

- 1..... ;
- 2..... ;
- 3..... ;
- 4..... ;

The tooth is considered as a beam with a profiled contour embedded in the gear crown and loaded with the normal force F_n .

Assumptions:

- the force is applied to the tip of the tooth and is taken only by one tooth;
- the width of the tooth at its base is S_F and has the length b .

The force F_n is translated in the direction of the action line until the intersection with the tooth axis of symmetry and is broken down into the tangential force F_{ta} and the radial force F_{ra} which produce a compound stress at the base of the tooth. The area of tooth breakage is highlighted in the figure 6.7.

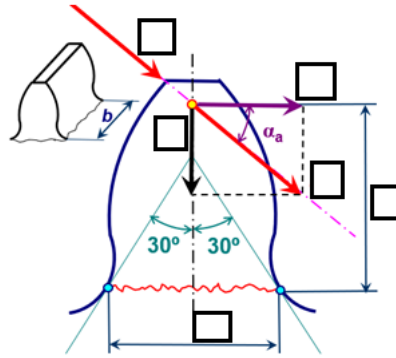


Fig. 6.7 Tooth breakage at bending stress

This area being stressed by tensile and taking into account a covering calculation but at the same time neglecting compression, it is obtained:

The maximum bending stress:

$$\sigma_F = \frac{M}{W} = \frac{F_{n1_{ef}} \cdot \cos \alpha_a}{b} \cdot \frac{6 \cdot h_F}{S_F^2}$$

$$\sigma_F = \frac{2 \cdot T_1 \cdot C_F \cdot \cos \alpha_a}{d_{w1} \cdot b \cdot \cos \alpha_w} \cdot \frac{6 \cdot h_F}{S_F^2} = \frac{2 \cdot T_1 \cdot C_F}{d_{w1} \cdot b \cdot m} \cdot \frac{\cos \alpha_a}{\cos \alpha_w} \cdot \frac{6 \cdot \frac{h_F}{m}}{\frac{S_F^2}{m^2}}$$

If the introduction of correction factors is taken into account, the expression for the bending stress becomes:

$$\sigma_F = \frac{2 \cdot T_1 \cdot C_F}{d_{w1} \cdot b \cdot m} \cdot Y_{Fa} \cdot Y_{Sa} \cdot Y_\epsilon \leq \sigma_{FP},$$

where $\sigma_{FP} - \dots\dots\dots$;
 $Y_{Fa} - \dots\dots\dots$;
 $Y_{Sa} - \dots\dots\dots$;
 $Y_{\varepsilon} - \dots\dots\dots$;
 $\dots\dots\dots$.

Dimensioning the cylindrical spur gear

If the modulus is considered to be:

$$m = \frac{d_{w1} \cos \alpha_w}{z_1 \cos \alpha} = \frac{2 \cdot a_w \cos \alpha_w}{z_1 \cdot (1+u) \cos \alpha} ,$$

and if the permissible stress is:

$$\sigma_{FP} = \frac{2 \cdot T_1 \cdot C_F}{\frac{2 \cdot a_w}{1+u} \cdot \psi_a \cdot a_w \cdot \frac{2 \cdot a_w \cos \alpha_w}{z_1 \cdot (1+u) \cos \alpha}} \cdot Y_{Fa} \cdot Y_{Sa} \cdot Y_{\varepsilon} ,$$

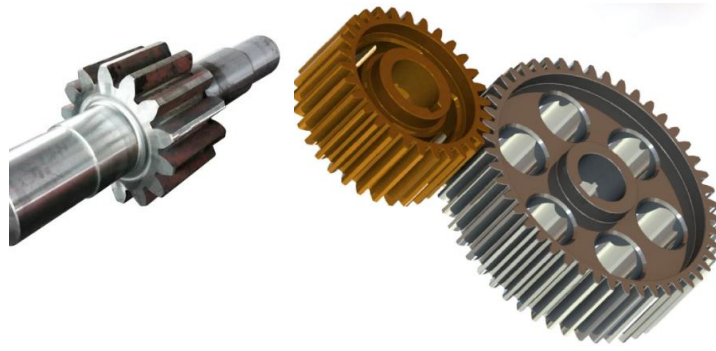
but also:

$$\sigma_{FP} = \frac{T_1 \cdot C_F \cdot z_1 \cdot (1+u)^2}{2 \cdot \psi_a \cdot a_w^3} \cdot \frac{\cos \alpha}{\cos \alpha_w} \cdot Y_{Fa} \cdot Y_{Sa} \cdot Y_{\varepsilon} ,$$

then the calculation relationship of the required center distance can be determined, based on the bending stress, thus:

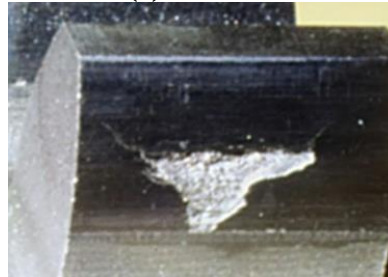
$$a_{w_nec} = \sqrt[3]{\frac{T_1 \cdot C_F \cdot z_1 \cdot (1+u)^2}{2 \cdot \psi_a \cdot \sigma_{FP}} \cdot \frac{\cos \alpha}{\cos \alpha_w} \cdot Y_{Fa} \cdot Y_{Sa} \cdot Y_{\varepsilon}} .$$

In figure 6.8 (a) a cylindrical spur pinion is presented, (b) a cylindrical spur gear; (c) "Pitting" type wear on the tooth flank; (d) breaking the tooth at the base; (e) breaking the tooth at bending; (f) traces of corrosion on the teeth.

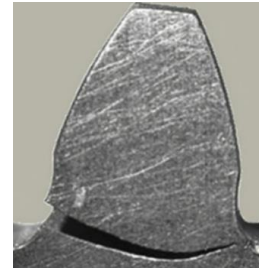


(a)

(b)



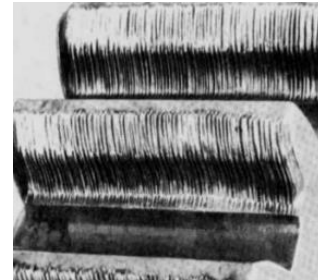
(c)



(d)



(e)



(f)

Fig. 6.8 Examples of gearwheels, gears and damage to spur gear teeth

[illegible]

[illegible]

7. CYLINDRICAL HELICAL GEARS

Forces in cylindrical helical gears

Figure 7.1 shows the simplified diagram of the representation of the forces in cylindrical helical gears.

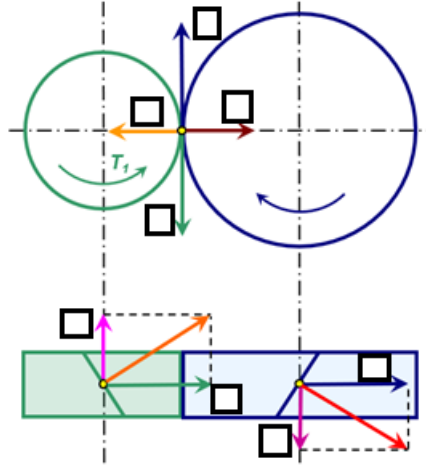


Fig. 7.1 Forces in Cylindrical Helical Gears

Knowing the expression of the tangential force in the pinion: $F_{t1} = \frac{2 \cdot T_1}{d_{w1}}$,

$$F_{n1} = \frac{F_{b1}}{\cos \beta_b} = \frac{F_{t1}}{\cos \alpha_{wt} \cos \beta_b} = \frac{2 \cdot T_1}{d_{w1} \cos \alpha_{wt} \cos \beta_b} \quad (\text{from gearing plane}), \text{ and}$$

$$F_{n1} = \frac{F_{01}}{\cos \alpha_{wn}} = \frac{F_{t1}}{\cos \alpha_{wn} \cos \beta_w} \quad (\text{from the operating plane}), \text{ as in figure 7.2, is}$$

determined:

Force: $F_{a1} = F_{t1} \tan \beta_w$

Force: $F_{r1} = F_{t1} \tan \alpha_{wt}$.

In accordance with the Equal and opposite reaction principle:

$$F_{t1} = F_{t2} \quad , \quad F_{n1} = F_{n2} \quad , \quad F_{a1} = F_{a2} \quad , \quad F_{r1} = F_{r2}$$

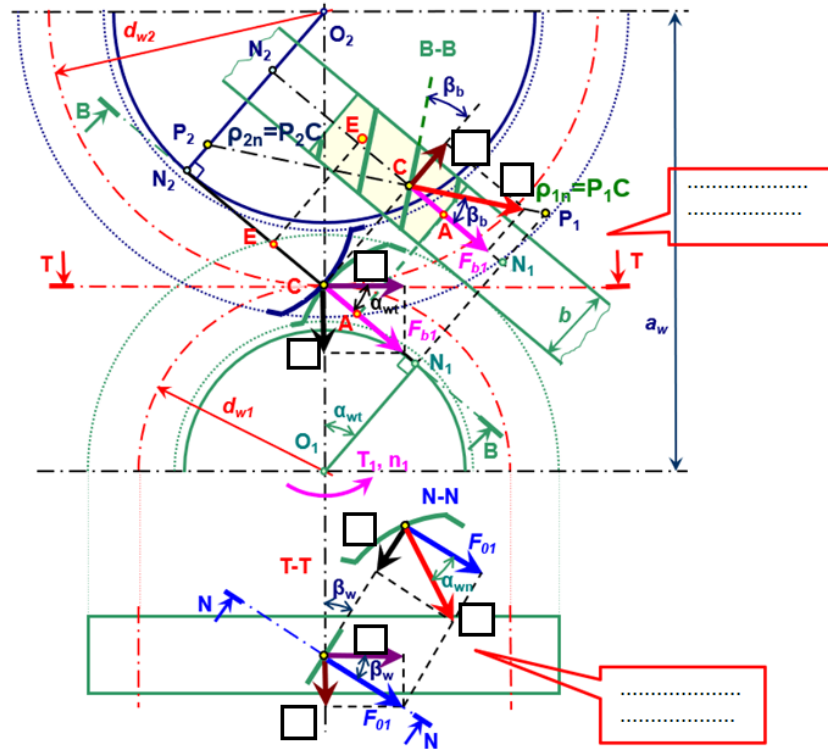


Fig. 7.2 Forces in the specific plans of helical gears

Geometric elements

In case of cylindrical helical gears, the module will standardize in the.....plane, being noted as m_n . The module noted as m_t is.....:

$$m_t = \frac{m_n}{\cos \beta}$$

The symbol a represents.....

and its formula is:
$$a = \frac{m_t \cdot (z_1 + z_2)}{2} = \frac{m_n (z_1 + z_2)}{2 \cos \beta}.$$

Angle α_t is..... and

is determined as:
$$\tan \alpha_t = \frac{\tan \alpha_n}{\cos \beta}$$

The relationship between “ a ”

and is: $a_w \quad a \cdot \cos \alpha_t = a_w \cdot \cos \alpha_{wt}.$

The diameters: $d_{1,2} = m_t \cdot z_{1,2} = \frac{m_n \cdot z_{1,2}}{\cos \beta}.$

The diameters: $d_{b1,2} = d_{1,2} \cdot \cos \alpha_t$

The diameters

$$d_{w1,2} = d_{1,2} \cdot \frac{\cos \alpha_t}{\cos \alpha_{wt}} = m_t \cdot z_{1,2} \cdot \frac{\cos \alpha_t}{\cos \alpha_{wt}} = \frac{m_n}{\cos \beta} \cdot z_{1,2} \cdot \frac{\cos \alpha_t}{\cos \alpha_{wt}}.$$

The distance: $a_w = \frac{d_{w1} + d_{w2}}{2}.$

The diameters: $d_{a1,2} = \frac{m_n}{\cos \beta} \cdot \left[z_{1,2} + 2 \cdot (h_{an}^* + x_{n1,2} - \Delta y_n) \cdot \cos \beta \right]$

The diameters.....: $d_{f1,2} = m_n \cdot \left[\frac{z_{1,2}}{\cos \beta} - 2 \cdot (h_{an}^* + c_n^* - x_{n1,2}) \right]$

In case of helical gears differ:

ε_α –

$$\varepsilon_\alpha = \frac{\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2} - 2a_w \sin \alpha_{wt}}{2\pi m_n \cos \alpha_t} \cdot \cos \beta$$

$\varepsilon_\beta = \dots\dots\dots$
 $\dots\dots\dots$

$$\varepsilon_\beta = \frac{b \cdot \sin \beta}{\pi \cdot m_n}$$

$\varepsilon_\gamma = \dots\dots\dots$

$$\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta$$

Strength calculation of cylindrical helical gear

The geometric calculation is carried out in the..... plane, and the strength calculation is carried out in the..... plane, where the minimum strength of the tooth is obtained.

A cylindrical helical gear can be replaced with an equivalent spur gear (figure 7.3). This is also called as..... and it has the numbers of teeth z_{v1} and z_{v2} , module m_n , and the shift of profile coefficients x_{n1} and x_{n2} .

The N-N plane intersects the pitch cylinder after an ellipse and the engagement takes place on a portion of the ellipse corresponding to 2 ... 3 normal pitches; the flanks of the teeth are considered to belong to a cylindrical gear with the radius of the pitch circle equal to the radius of curvature of the ellipse at point C.

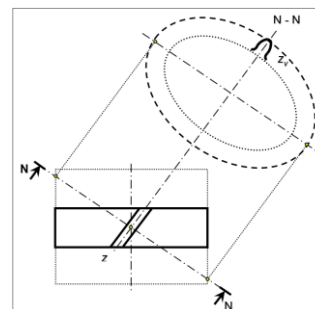


Fig. 7.3 Equivalent gear

The ellipse curvature radius at point C (figure 7.4) is: $\rho_v = \frac{a^2}{b} = \frac{d}{2 \cos^2 \beta}$

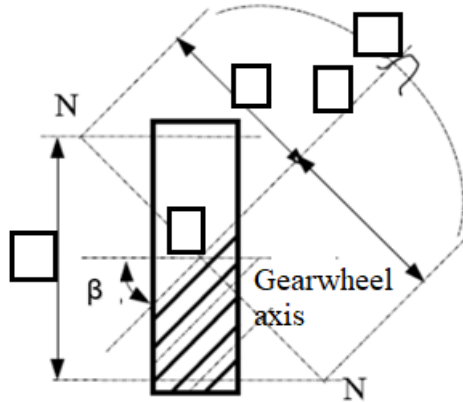


Fig. 7.4 Ellipse curvature radius

a – ;

b – ;

d_v – ;

..... ;

z_v – ;

..... ;

z – ;

..... .

If: $d_v = d / \cos^2 \beta$, $d_v = m_n z_v = m_t \cos \beta \cdot z_v$, $m_t \cos \beta \cdot z_v = \frac{m_t z}{\cos^2 \beta}$,

it results:

$$z_{v1,2} = \frac{z_{1,2}}{\cos^3 \beta} .$$

Contact pressure stress calculation

Hertz's relation is used:

$$\text{Contact pressure stress: } \sigma_H = \sqrt{\frac{F_n}{L_k} \cdot \frac{1}{\rho_n} \cdot \frac{1}{\pi \cdot \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)}}$$

$$\text{Effective normal force: } F_n = F_{n1_ef} = \frac{2 \cdot T_1 \cdot C_H}{d_{w1} \cdot \cos \alpha_t \cdot \cos \beta_b}$$

Contact length: $L_k = \frac{b \cdot \varepsilon_\alpha}{\cos \beta_b}$

Raza de curbură redusă $\frac{1}{\rho_n} = \frac{2 \cos \beta_b}{d_{w1} \sin \alpha_{wt}} \cdot \frac{u+1}{u}$

Therefore, the contact pressure stress becomes:

$$\begin{aligned} \sigma_H &= \sqrt{\frac{2 \cdot T_1 \cdot C_H}{d_{w1} \cos \alpha_{wt} \cos \beta_b} \cdot \frac{\cos \beta_b}{b \cdot \varepsilon_\alpha} \cdot \frac{2 \cos \beta_b}{d_{w1} \sin \alpha_{wt}} \cdot \frac{u+1}{u} \cdot \frac{1}{\pi \cdot \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)}} = \\ &= \sqrt{\frac{1}{\pi \cdot \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)}} \cdot \sqrt{\frac{2 \cos \beta_b}{\sin \alpha_{wt} \cos \alpha_{wt}}} \cdot \sqrt{\frac{1}{\varepsilon_\alpha}} \cdot \sqrt{\frac{2 \cdot T_1 \cdot C_H}{b \cdot d_{w1}^2} \cdot \frac{u+1}{u}} \end{aligned}$$

Taking into account the definition of material, contact, and degree of coverage factors, the relationship to the contact pressure stress checking becomes:

$$\sigma_H = Z_E \cdot Z_H \cdot Z_\varepsilon \cdot \sqrt{\frac{2 \cdot T_1 \cdot C_H}{b \cdot d_{w1}^2} \cdot \frac{u+1}{u}} \leq \sigma_{HP}.$$

The dimensioning of the cylindrical helical gear, at contact pressure stress, is made by:

$$a_{w_nec} = (1+u) \cdot \sqrt[3]{\frac{T_1 \cdot C_H \cdot Z_E^2 \cdot Z_H^2 \cdot Z_\varepsilon^2}{2 \cdot \psi_d \cdot u \cdot \sigma_{HP}^2}}$$

Bending stress calculation

The gear-checking relationship for the maximum bending stress is:

$$\sigma_F = \frac{2 \cdot T_1 \cdot C_F}{d_{w1} \cdot b \cdot m_n} \cdot Y_{Fa} \cdot Y_{Sa} \cdot Y_\varepsilon \cdot Y_\beta \leq \sigma_{FP}, \text{ where:}$$

Y_{Fa} — ;

Y_{Sa} — ;

Y_{ε} — ;

Y_{β} —

The dimensioning of the cylindrical helical gear, at bending stress, is made by:

$$a_{w_nec} = \sqrt[3]{\frac{T_1 \cdot C_F \cdot (1+u)^2 \cdot z_1}{2\psi_a \cdot \sigma_{FP}}} \cdot \frac{\cos \alpha_t}{\cos \beta \cos \alpha_{wt}} \cdot Y_{Fa} \cdot Y_{Sa} \cdot Y_{\varepsilon} \cdot Y_{\beta} .$$

Figure 7.5 shows some examples of cylindrical helical gearwheels and gears made by them.



(a)



(b)



(c)

Fig. 7.5 (a) Cylindrical helical pinion; (b) Cylindrical helical gear;
(c) Double helical gear

[illegible]

[illegible]

8. BEVEL GEARS. STRAIGHT BEVEL GEARS

Bevel gears have intersecting coplanar axes and the rolling surfaces form a pair of tangent cones that roll without slipping.

Bevel gears have a number of advantages such as:

- ✓;
- ✓;
- ✓;
- ✓;
- ✓

According to the shape of the teeth in the case of bevel gear: straight, inclined, in an involute arch, in a circular arch (figure 8.1).

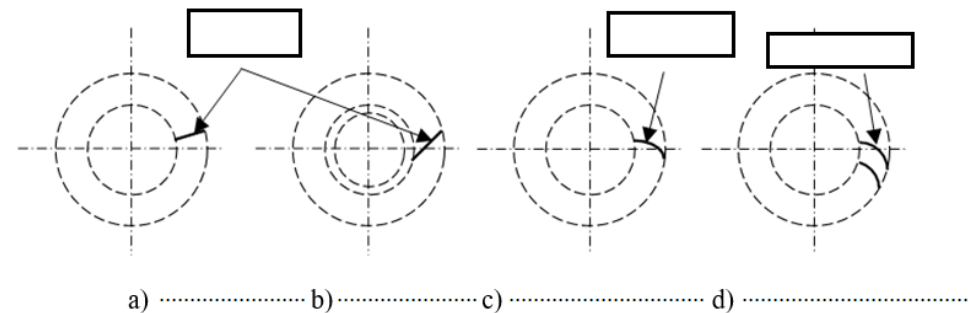


Fig. 8.1 Example of bevel gear teeth shapes

Calculations regarding the kinematics, the degree of coverage and the straightness of the bevel gears are carried out using the replacement gear (also called virtual gear), corresponding to the medium backcones.

The geometric elements corresponding to the equivalent gear are noted by the index v.

The main geometric elements of the virtual bevel gear are shown in the figure 8.2.

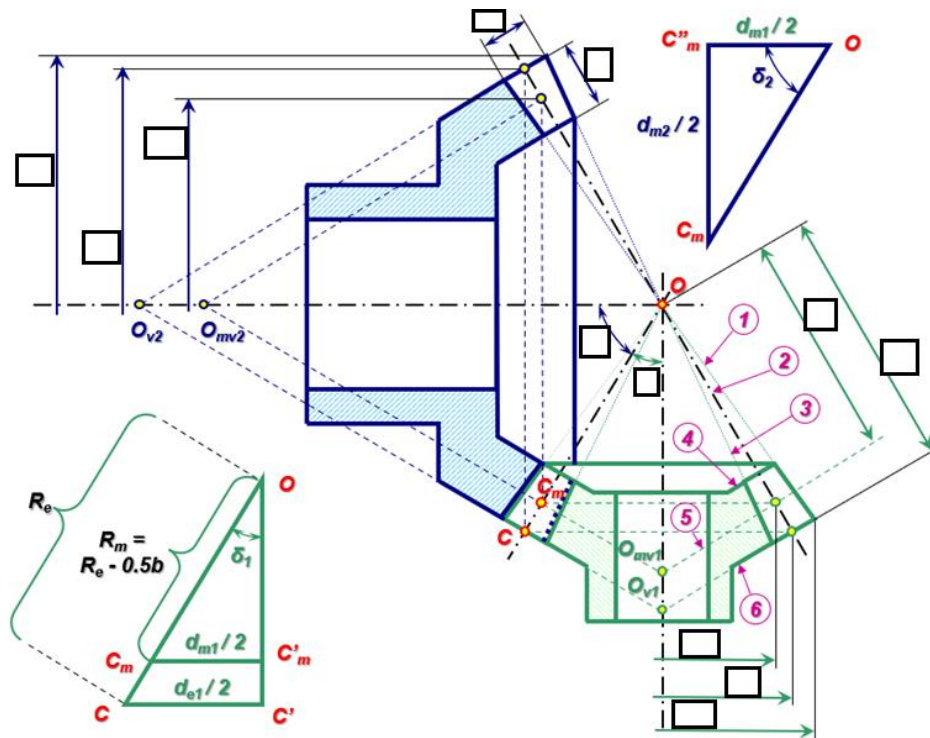


Fig. 8.2. The geometric elements of the virtual bevel gear

$$\text{Diameters} \dots\dots\dots: d_{v1,2} = \frac{d_{m1,2}}{\cos \delta_{1,2}}$$

$$\text{Distance} \dots\dots\dots: a_v = \frac{d_{v1} + d_{v2}}{2}.$$

The replacement gear, having the same m_{mt} module as the bevel gear, is written:

$$m_{mt} z_{v1,2} = \frac{m_{mt} z_{1,2}}{\cos \delta_{1,2}},$$

where: $z_{v1,2}$

$z_{1,2}$

Thus, the relationship between the number of teeth is:

$$z_{v1,2} = \frac{z_{1,2}}{\cos \delta_{1,2}} \quad u_v = \frac{z_{v2}}{z_{v1}} = \frac{z_2}{z_1} \frac{\cos \delta_1}{\cos \delta_2} = u^2$$

Notations: Cone: “a”, Cone “-”,

Cone “f”, Cone “i”,

Cone “m”, ... “e”,

- whole depth

$$h = m_e \cdot (2 \cdot h_a^* + c^*)$$

- angle....., [°]

$$\tan \delta_1 = \frac{z_1}{z_2},$$

- angle....., [°]

$$\delta_2 = 90^\circ - \delta_1,$$

- length.....[mm]

$$R_e = \frac{d_{e1,2}}{2 \cdot \sin \delta_{1,2}} = \frac{m_e \cdot z_{1,2}}{2 \cdot \sin \delta_{1,2}}$$

- coefficients.....

$$x_{r1} = -x_{r2} \quad x_{r1} = -x_{r2};$$

- diameters.....,[mm]

$$d_{a1,2} = m_e \cdot \left[z_{1,2} + 2 \cdot (h_a^* + x_{r1,2}) \cdot \cos \delta_{1,2} \right]$$

- diameters.....,[mm]

$$d_{f1,2} = m_e \cdot \left[z_{1,2} - 2 \cdot (h_a^* + c^* - x_{r1,2}) \cdot \cos \delta_{1,2} \right]$$

- diameters.....,[mm]

$$d_{m1} = \frac{2R_e(1 - 0,5\psi_R)}{\sqrt{1 + u_I^2}}, \quad d_{m2} = \frac{2R_e(1 - 0,5\psi_R)u_I}{\sqrt{1 + u_I^2}}$$

- diameters.....,[mm]

$$d_{e1,2} = m_e \cdot z_{1,2}$$

- diameters.....,[mm]

$$d_{av1} = d_{v1} + 2h_{a1}; \quad d_{av2} = d_{v2} + 2h_{a2}$$

- diameters.....,[mm]

$$d_{bv1} = d_{v1} \cos(\alpha); \quad d_{bv2} = d_{v2} \cos(\alpha)$$

- degree of.....

$$\varepsilon_\alpha = \frac{\sqrt{d_{av1}^2 - d_{bv1}^2} + \sqrt{d_{av2}^2 - d_{bv2}^2} - 2a_v \sin(\alpha)}{2 \cdot \pi \cdot m_e \cos(\alpha)}$$

- coefficient.....

$$\psi_R = \frac{b}{R_e} \Leftrightarrow b = \psi_R \cdot R_e$$

- angle.....,[°]

$$\theta_{a1,2} = \arctan\left(\frac{h_{a1,2}}{R}\right)$$

- angle....., [°]

$$\theta_{f1,2} = \arctan\left(\frac{h_{f1,2}}{R}\right)$$

- angles....., [°]

$$\delta_{a1,2} = \delta_{1,2} + \theta_{a1,2}$$

- angles....., [°]

$$\delta_{f1,2} = \delta_{1,2} - \theta_{f1,2}$$

Forces in bevel gear

The way forces are distributed in the bevel gear is represented in figure 8.3.

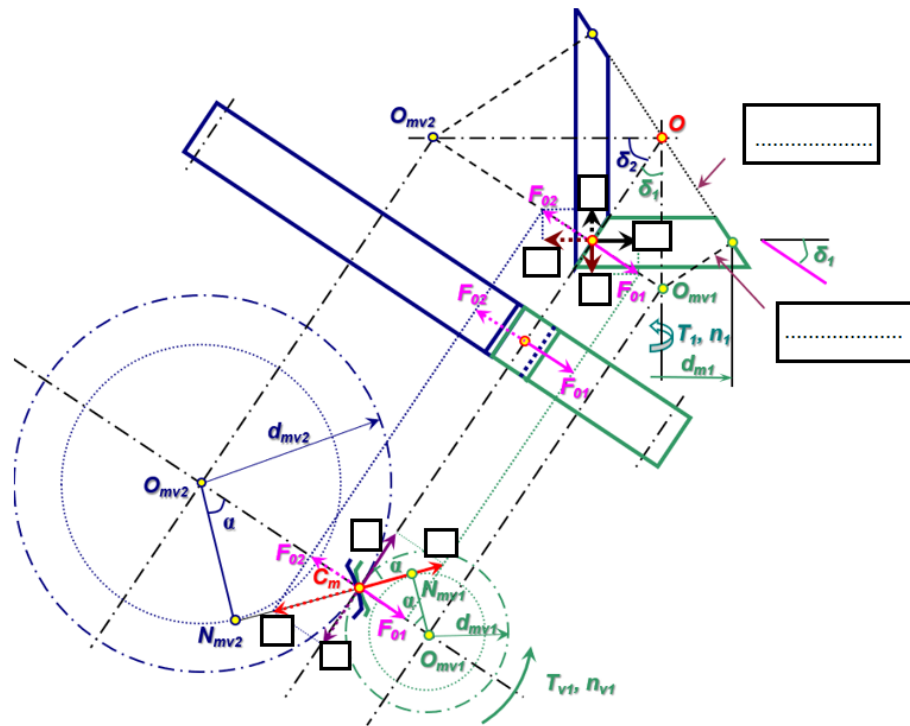


Fig. 8.3 Forces distribution in bevel gear

From the torsion torque relation:

$$T_1 = F_{t1} \cdot \frac{d_{m1}}{2} \Rightarrow ,$$

Force..... $F_{t1} = \frac{2T_1}{d_{m1}}$, and

Force..... $F_{n1} = \frac{F_{t1}}{\cos \alpha} = \frac{2T_1}{d_{m1} \cos \alpha}$

Force..... $F_{r1} = F_{o1} \cos \delta_1 = F_{t1} \tan \alpha \cos \delta_1$,

Force..... $F_{a1} = F_{o1} \sin \delta_1 = F_{t1} \tan \alpha \sin \delta_1$,

also: $F_{t2} = F_{t1}$ $F_{n2} = F_{n1}$ $F_{r2} = F_{a1}$ $F_{a2} = F_{r1}$.

Calculation of bevel gears at contact pressure

It is adopted Hertz relation: $\sigma_H = \sqrt{\frac{F_n}{L_k} \cdot \frac{1}{\rho_{mv}} \cdot \frac{1}{\pi \cdot \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)}}$.

Are known:

Force.....: $F_n = F_{n1_ef} = \frac{2 \cdot T_1 \cdot C_H}{d_{m1} \cos \alpha}$

Length.....: $L_k = \frac{b}{\frac{4 - \varepsilon_{\alpha mv}}{3}}$

Radii.....:

$$\rho_{mv1} = C_m N_{mv1} = \frac{d_{m1} \sin \alpha}{2 \cos \delta_1}$$

$$\rho_{mv2} = C_m N_{mv2} = \frac{d_{m2} \sin \alpha}{2 \cos \delta_2}$$

$$\frac{1}{\rho_{mv}} = \frac{1}{\rho_{mv1}} + \frac{1}{\rho_{mv2}} = \frac{2 \cdot \sqrt{1+u^2}}{d_{m1} \cdot u \cdot \sin \alpha}$$

The contact pressure becomes:

$$\sigma_H = \sqrt{\frac{2 \cdot T_1 \cdot C_H}{d_{m1} \cos \alpha} \cdot \frac{4 - \varepsilon_{\alpha mv}}{3} \cdot \frac{2 \cdot \sqrt{1 + u^2}}{d_{m1} \cdot u \cdot \sin \alpha} \cdot \frac{1}{\pi \cdot \left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)}} =$$

$$= \sqrt{\frac{1}{\pi \cdot \left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)}} \cdot \sqrt{\frac{2}{\sin \alpha \cos \alpha}} \cdot \sqrt{\frac{4 - \varepsilon_{\alpha mv}}{3}} \cdot \sqrt{\frac{2 \cdot T_1 \cdot C_H}{b \cdot d_{m1}^2} \cdot \frac{\sqrt{1 + u^2}}{u}}$$

The relationship for checking at contact pressure strength:

$$\sigma_H = Z_E \cdot Z_{Hv} \cdot Z_{\varepsilon v} \cdot \sqrt{\frac{2 \cdot T_1 \cdot C_H}{b \cdot d_{m1}^2} \cdot \frac{\sqrt{1 + u^2}}{u}} \leq \sigma_{HP}$$

Dimensioning the bevel gear at contact pressure

If: $d_{m1} = \frac{2R_e \cdot (1 - 0.5 \cdot \psi_R)}{\sqrt{1 + u^2}}$ and

$$d_{m1}^2 \approx \frac{4R_e^2 \cdot (1 - \psi_R)}{1 + u^2}, \text{ iar } b = \psi_R \cdot R_e,$$

It is obtained.....:

$$\sigma_{HP} = Z_E \cdot Z_{Hv} \cdot Z_{\varepsilon v} \cdot \sqrt{\frac{2 \cdot T_1 \cdot C_H}{\psi_R \cdot R_e \cdot \frac{4R_e^2 \cdot (1 - \psi_R)}{1 + u^2}} \cdot \frac{\sqrt{1 + u^2}}{u}}.$$

If: $\sigma_{HP}^2 = Z_E^2 \cdot Z_{Hv}^2 \cdot Z_{\varepsilon v}^2 \cdot \frac{T_1 \cdot C_H}{2 \cdot R_e^3 \cdot \psi_R \cdot (1 - \psi_R)} \cdot \frac{(\sqrt{1 + u^2})^3}{u},$

it can be determined the length.....

.....

, based on contact pressure stress, thus:

$$R_{e_nec} = \sqrt{1+u^2} \sqrt[3]{\frac{T_1 \cdot C_H \cdot Z_E^2 \cdot Z_{Hv}^2 \cdot Z_{\varepsilon v}^2}{2 \cdot u \cdot \psi_R \cdot (1-\psi_R) \cdot \sigma_{HP}^2}}$$

Calculation the bevel gear at bending

Maximum bending stress is:

$$\sigma_F = \frac{2 \cdot T_1 \cdot C_F}{d_{m1} \cdot b \cdot m_m} \cdot Y_{Fa} \cdot Y_{Sa} \cdot Y_{\varepsilon} \leq \sigma_{FP}, \text{ where:}$$

Y_{Fa} – ;

Y_{Sa} – ;

Y_{ε} –

Dimensioning the bevel gear at bending

Being known that: $d_{m1} \cdot m_m = \frac{4R_e^2 \cdot (1-\psi_R)}{z_1 \cdot (1+u^2)}$, iar $b = \psi_R \cdot R_e$,

It is obtained:

$$\sigma_{FP} = \frac{2 \cdot T_1 \cdot C_F}{\psi_R \cdot R_e \cdot \frac{4R_e^2 \cdot (1-\psi_R)}{z_1 \cdot (1+u^2)}} \cdot Y_{Fa} \cdot Y_{Sa} \cdot Y_{\varepsilon},$$

thus:

$$\sigma_{FP} = \frac{T_1 \cdot C_F \cdot Y_{Fa} \cdot Y_{Sa} \cdot Y_{\varepsilon} \cdot z_1 \cdot (1+u^2)}{2R_e^3 \cdot \psi_R \cdot (1-\psi_R)}, \text{ so}$$

it can be determined the length.....

.....,

based on bending, thus:

$$R_{e_nec} = \sqrt[3]{\frac{T_1 \cdot C_F \cdot Y_{Fa} \cdot Y_{Sa} \cdot Y_\varepsilon \cdot Z_1 \cdot (1+u^2)}{2 \cdot \psi_R \cdot (1-\psi_R) \cdot \sigma_{FP}}}$$

In figure 8.4 some examples of bevel gears are presented.



(a)



(b)



(c)



(d)

Fig. 8.4 (a) Bevel pinion; (b) Spiral teeth bevel pinion;
(c) Bevel gear; (d) Spiral bevel gear.

[illegible]

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9. WORM GEARS

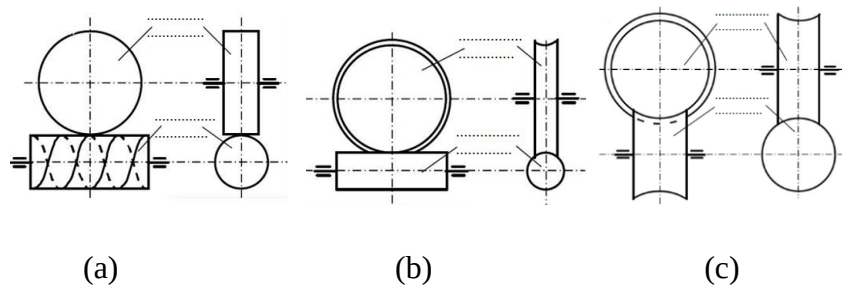
General considerations.

The number of teeth on a worm is also called The geometric elements are calculated as in the case of helical gears. The difference lies in the globoidal shape of the worm wheel, to enclose the worm and thus achieve linear contact.

Advantages	Disadvantages
.....	
.....	
.....	
.....	
.....	
.....	
.....	
.....	
.....	
.....	

Figure 9.1 shows different types of worm gears, as follows:

- a).....; b).....;
c)..... .
-



(a) (b) (c)

Fig. 9.1 Types of worm gears

Cylindrical worm gears.

A reference rack cannot define the worm gear teeth as it can with spur gears; thus, according to ISO 10064-2 (figure 9.2), a cylindrical reference worm was adopted.

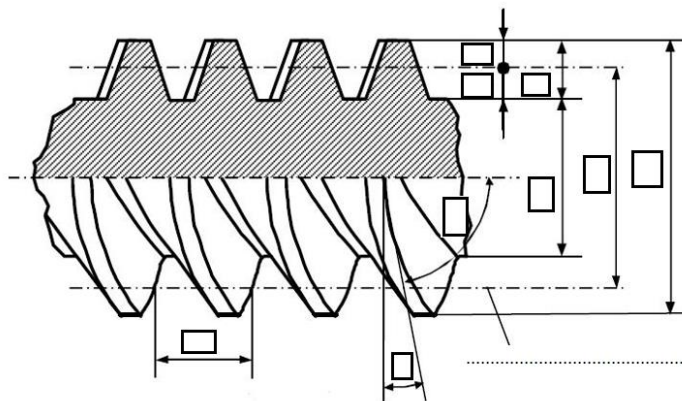


Fig. 9.2 Cylindrical reference worm

α_x —

y —

p_x —

d_1 —

..... ;
 $\beta_1 -$;
 $d_a, d_f, d_0 -$
 ;
 $h_a, h_f, h -$
 ;

According to **ISO 10064**, at worm gears, the axial module is standardized, m_x . The recommended values are presented in table 9.1.

Tab. 9.1 Axial modules range

The modules range from cylindrical worm gears m_x , mm									
0,1	0,125	0,16	0,2	0,25	0,315	0,4	0,5	0,63	0,8
1	1,25	1,6	2	2,5	3,15	4	5	6,3	8
10	12,5	16	20	25	—	—	—	—	—

According to ISO 10064 are indicated because: $d_1 = m_x \cdot q$, for each module m_x three values of the parameter specific to worm gears (table 9.2), called....., q .

Tab. 9.2 Diametral coefficient values

Diametral coefficient values, ISO 10064							
m_x	1...1,6	2 and 2,5	3...4	5...6,3	7...10	12...16	20 and 25
q	12	10	10	9	9	8	7
	14	12	11	10	10	9	8
	16	14	12	12	11	10	9

Geometric elements of the worm gear

In figure 9.3 are the main geometric elements of the worm gear.

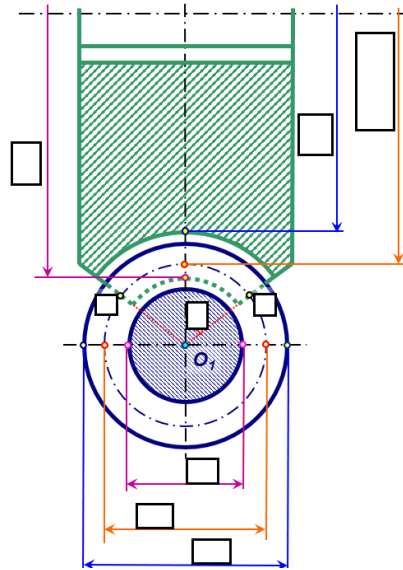


Fig. 9.3 Worm gear geometry

$$d_1 = m_x \cdot q$$

$$d_{w1} = m_x \cdot (q + 2x)$$

Worm: $d_{a1} = m_x \cdot (q + 2h_a^*) = m_x \cdot (q + 2)$

$$d_{f1} = m_x \cdot (q - 2h_a^* - 2c^*) = m_x \cdot (q - 2.5)$$

$$d_2 = d_{w2} = m_x \cdot z_2$$

$$d_{a2} = m_x \cdot (z_2 + 2h_a^* + 2x) = m_x \cdot (z_2 + 2 + 2x)$$

Worm gear: $d_{f2} = m_x \cdot (z_2 - 2h_a^* - 2c^* + 2x) = m_x \cdot (z_2 - 2.5 + 2x)$

$$d_{e2} = d_{a2} + \frac{6 \cdot m_x}{z_1 + 2}$$

d_e -;

γ -;

$$\gamma = \arctg\left(\frac{z_1}{q}\right)$$

a -

$$a = \frac{m_x \cdot (q + z_2)}{2}$$

a_w -

$$a_w = \frac{d_{w1} + d_{w2}}{2}$$

m_{x_nec} -

$$m_{x_nec} = \frac{2 \cdot a_w}{q + z_2}$$

h -

$$h = m_x \cdot (2 \cdot h_a^* + c^*)$$

z -

for z≤3: $b \leq 0,75 \cdot d_{a1}$, for z=4: $b \leq 0,67 \cdot d_{a1}$

L -

for z=1,2: $L = (11 + 0,06 \cdot z_2) \cdot m_x$, for z=3,4: $L = (12,5 + 0,09 \cdot z_2) \cdot m_x$.

The materials used to manufacture the worm are:
..... and for
the worm gear
.....

Forces in worm gear

In figure 9.4 the distribution of forces in the cylindrical worm gear is presented.

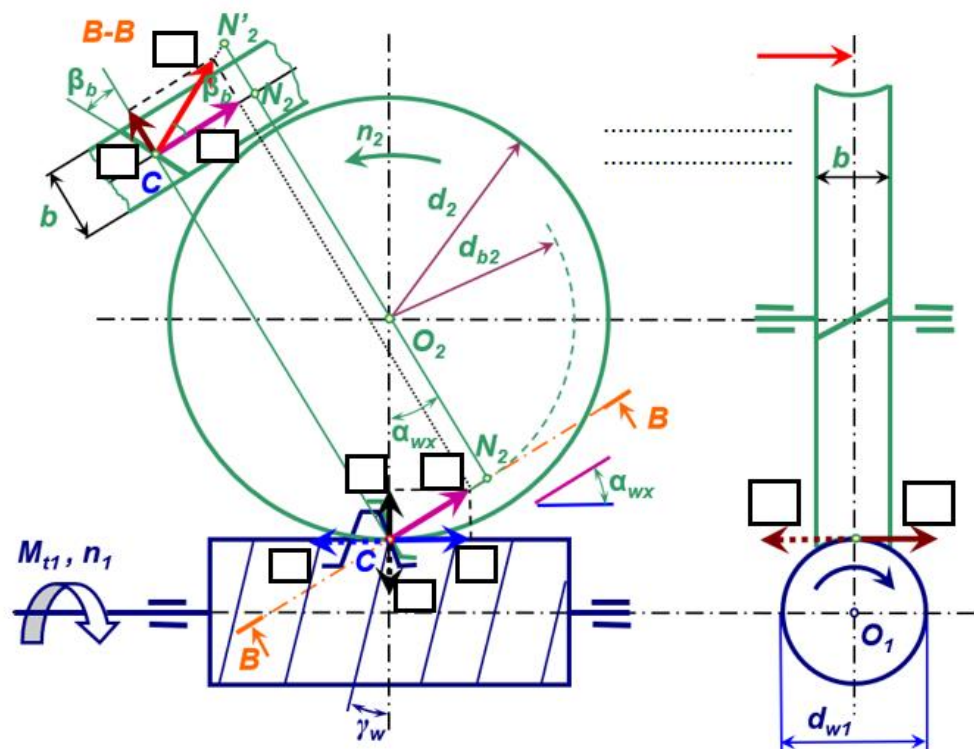


Fig. 9.4 Forces in the cylindrical worm gear

It is known: $F_{t2} = \frac{2 \cdot T_2}{d_2} = F_{a1}$, and: $F_{r2} = F_{t2} \cdot \tan \alpha_{wx} = F_{r1}$, and

$$F_{n2} = \frac{2 \cdot T_2}{d_2 \cos \alpha_{wx} \cos \beta_b} = F_{n1}.$$

The worm can be assimilated to a screw, so the calculation relationship for the tangential force F_{t1} can be written as in the case of the threaded assemblies:

$$F_{a2} = F_{t2} \tan(\gamma_w + \phi') = F_{t1}$$

Contact pressure stress calculation:

Starting with Hertz relation: $\sigma_H = \sqrt{\frac{F_n}{L_k} \cdot \frac{1}{\rho_n} \cdot \frac{1}{\pi \cdot \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)}}$, where:

The effective normal force is: $F_n = F_{n2_ef} = \frac{2 \cdot T_2 \cdot C_H}{d_2 \cos \alpha_{wx} \cos \beta_b}$

Contact length: $L_k = d_1 \cdot \frac{\pi \cdot \chi}{180^\circ} \cdot \frac{\varepsilon_\alpha}{\cos \beta_b} \cdot \xi$, where:

χ -

ξ -

Reduced curvature radius: $\frac{1}{\rho_n} = \frac{2 \cos \beta_b}{d_2 \sin \alpha_{wx}}$.

Thus, the contact pressure stress becomes:

$$\sigma_H = \sqrt{\frac{1}{\pi \cdot \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)}} \cdot \sqrt{\frac{2 \cos \beta_b}{\sin \alpha_{wx} \cos \alpha_{wx}}} \cdot \sqrt{\frac{180^\circ}{\chi \cdot \pi \cdot \xi \cdot \varepsilon_\alpha}} \cdot \sqrt{\frac{2 \cdot T_2 \cdot C_H}{d_1 \cdot d_2^2}}$$

Taking into account the occurrence/using the material or contact and contact ratio factors, the strength checking relationship becomes:

$$\sigma_H = Z_E \cdot Z_H \cdot Z_\varepsilon \cdot Z_c \cdot \sqrt{\frac{2 \cdot T_2 \cdot C_H}{d_1 \cdot d_2^2}} \leq \sigma_{HP}.$$

Dimensioning the worm gear at contact pressure

The center distance:

$$a_w = \frac{m_x \cdot (q + z_2 + 2x)}{2}, \text{ where: } m_x = \frac{2a_w}{q + z_2 + 2x}$$

From the contact pressure checking relation is obtained:

$$\sigma_{HP}^2 = Z_E^2 \cdot Z_H^2 \cdot Z_\varepsilon^2 \cdot Z_c^2 \cdot \frac{T_2 \cdot C_H \cdot (q + z_2 + 2x)^3}{4 \cdot d_w^3 \cdot q \cdot z_2^2} \quad \text{and the center distance}$$

required for the contact pressure stress results:

$$a_{w_nec} = (q + z_2 + 2x) \cdot \sqrt[3]{Z_E^2 \cdot Z_H^2 \cdot Z_\varepsilon^2 \cdot Z_c^2 \cdot \frac{T_2 \cdot C_H}{4 \cdot q \cdot z_2^2 \cdot \sigma_{HP}^2}}.$$

Thermal calculation of the worm gear

Thermal calculation of the worm gear aimed to:

.....

The amount of heat released per unit of time is calculated as follows:

$$Q_1 = (1 - \eta) \cdot P_1 \cdot 10^3 \quad [W],$$

where: P_1 [kW] -,

and η -

The amount of heat discharged per unit of time, through the surface of the housing, is calculated with the relation:

$$Q_2 = \lambda \cdot (t - t_0) \cdot S \cdot (1 + \psi), \text{ where}$$

λ -

S [m²] -

$\Psi < 0,3$ -

t, t_0 [°C] -

Considering the operation in stationary mode, from the heat balance equation ($Q_1=Q_2$) results:

$$t = \frac{(1-\eta) \cdot P_1 \cdot 10^3}{\lambda \cdot (1+\psi) \cdot S} + t_0 \leq t_{\max}.$$

Bending stress calculation

The calculation of the maximum bending stress is performed for the worm gear tooth, its material having mechanical properties inferior to the material from which the worm is made.

The bending stress is:

$$\sigma_F = \frac{2 \cdot T_2 \cdot C_F}{m_x^3 \cdot q \cdot z_2} \cdot Y_F \cdot \frac{180^\circ}{\pi \cdot \chi \cdot \varepsilon_\alpha \cdot \xi} \cdot \frac{1}{\cos \gamma_w}, \text{ where:}$$

$$\frac{180^\circ}{\pi \cdot \chi \cdot \varepsilon_\alpha \cdot \xi} = Y_\varepsilon, \text{ and } \frac{1}{\cos \gamma_w} = Y_\gamma,$$

Y_F –

Y_ε –

Y_γ –

The bending stress relation becomes (for checking):

$$\sigma_F = \frac{2 \cdot T_2 \cdot C_F}{m_x^3 \cdot q \cdot z_2} \cdot Y_F \cdot Y_\varepsilon \cdot Y_\gamma \leq \sigma_{FP}.$$

Figure 9.5 (a) presents a worm shaft with a cylindrical worm, and (b) a one-piece bronze worm wheel. Figure 9.6 shows an example of a worm gear.



Fig. 9.5 (a) Worm shaft (b) Worm wheel



Fig. 9.6 Worm gear

[illegible]

10. Shafts

Shafts are:
.....
.....

In Figure 10.1, examples of shafts with mounted elements are presented. The input shaft with a pulley at the end, secured with a screw and washer, as well as bearings for support and a lip seal. The output shaft with a helical gear (mounted with a key), bearings for support, and a lipseal.

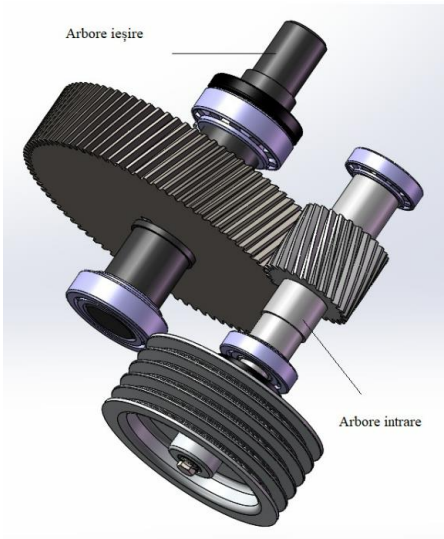


Fig. 10.1 Examples of shafts

Classification:

1. From the shape point of view:

- ;
- ;

2. From the rigidity point of view:

- ;
- ;
- ;

3. From the point of view of functionality:

- ;
- ;
- ;

4. From the point of view of position:

- ;
-

The materials from which the shafts are made are:

..... ;

Untreated shafts:..... ;

Treated shafts:

Design of shafts.

1. Dimensioning of shafts.

The following are known:

- ;
- ;
-

$$T(M_t) = 9550 \cdot \frac{P}{n},$$

where: T (M_t) - [N·m] ;

P - [kW] ;
n - [rotations/minute].

Pre-dimensioning:

The shaft has a standardized end (ISO 6412-1). Depending on the value of the torison torque, ... and (short series or long series) are chosen. The data in the table correspond to a material with a reference strength $\sigma_r = 550$ [MPa]. If the actual strength $\sigma_r \neq$ reference σ_r , a correction factor k is established.

$$M_{tc} = \frac{M_t}{k}; \quad unde \quad k = \frac{\sigma_{r \text{ efectiv}}}{\sigma_{r \text{ etalon}}}$$

The shaft DOES NOT have a standardized end; dimensioning is done by torsion.....

$$\tau_t = \frac{M_t}{W_p} = \frac{M_t}{\frac{\pi \cdot d^3}{16}} \quad , \quad d = \sqrt[3]{\frac{16 \cdot M_t}{\pi \cdot \tau_{at}}}$$

If the bending moment is unknown, a calculation approximation will be made, namely: $\tau_{at} = 12 \dots 25 \text{ MPa}$.

Design of shaft shapes. Shafts are simultaneously subjected to (by the transmitted torque) and to (by the forces introduced by the supported parts). An example of a shaft with different sections is presented in Figure 10.2.

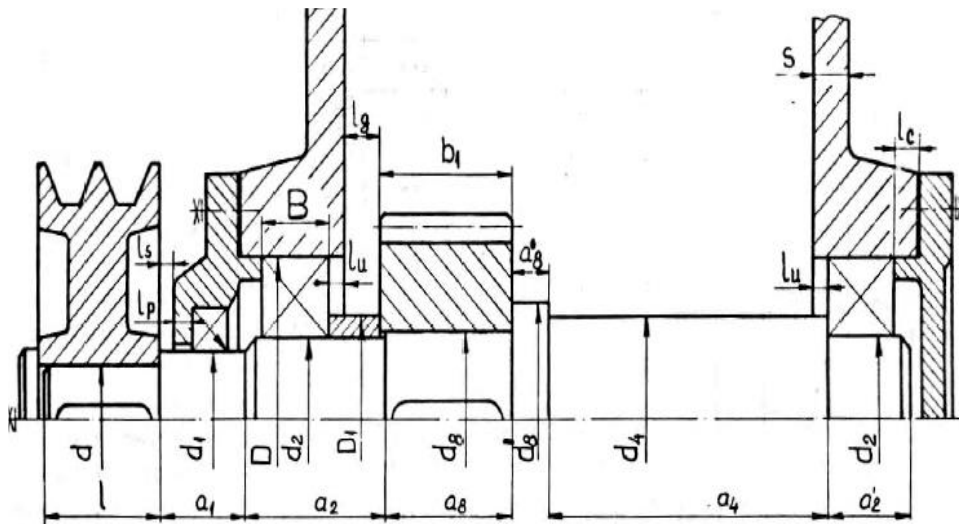


Fig. 10.2 Shaft sections

Section I:

- is called: ;
- from the pre-dimensioning stage, the diameter d_{CA} and length l_{CA} of this section are known.

Section II:

- is intended for: ;
- for the axial positioning of the pulley it is recommended that the diameter of the section be:

$$d_e = (1,15 \dots 1,2) \cdot d_{CA}$$

For centering the lip seal, it is recommended to: $l_{cen} \cong 0,5 \cdot \sqrt{D_r}$.

Section III:

- is intended for: ;

-
- the transmission of forces, as well as the axial positioning of the shaft, is achieved with the help of ;
 - The value of the dimension l_s'' is determined by the bearing lubrication system.

Section IV:

- is intended:..... ;
- the spacer ring must respect..... ;
- the distance from the reducer wall to the gear wheel

$$l_s''' = 5...10 \text{ mm}$$

- the hub length (for centering)

$$l_{butuc} = (0,6...1,2) \cdot d_{IV} [\text{mm}].$$

Section V:

- shoulder for:..... ;
- there are recommendations regarding its shape and dimensions.

Section VI:

- it's a section:..... ;
 - is dimensioned based on.....
-

Section VII:

- it is intended:..... ;
- if possible, a bearing is chosen
.....
.....

Securing the parts on the shaft end is done:
.....
.....

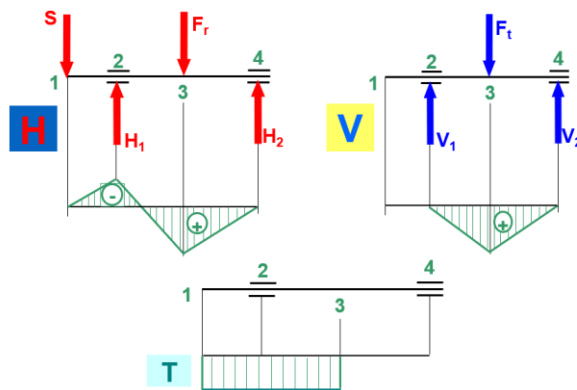
Verification of shafts.**1. Verification under combined loads (fig. 10.3)**

Fig. 10.3 Load diagram for shaft verification

Steps to follow::

- identify..... ;
- draw..... ;
- the resultant bending moments are calculated for each section by
..... ;

$$M_{iech} = \sqrt{M_{iH}^2 + M_{iV}^2}$$

- determine the value of the equivalent moment that takes into account
..... ;

$$M_{ech} = \sqrt{M_{iech}^2 + (\alpha \cdot T)^2}$$

where: α -

M_i -

T -

$$\alpha = \frac{\sigma_{ai III}}{\sigma_{ai II}},$$

$\sigma_{ai III}$ -

$\sigma_{ai II}$ - ;(fig. 10.4).

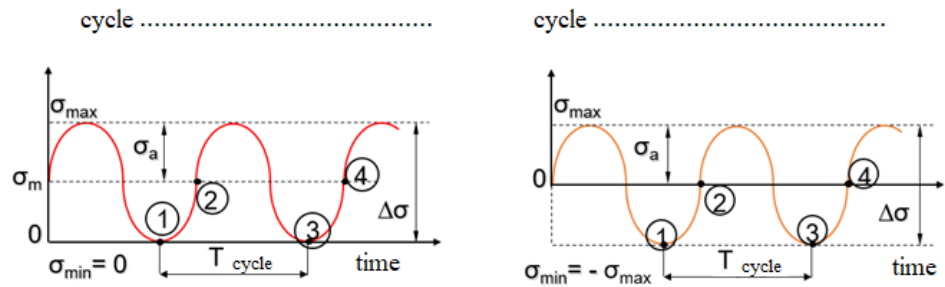


Fig. 10.4 Load cycles

Verification under combined loads is done using the following formula:

$$\sigma_{ech} = \frac{\sqrt{M_{iH}^2 + M_{iV}^2 + (\alpha \cdot T)^2}}{W} \leq \sigma_{ai II},$$

where – W

;

$$W = \frac{\pi \cdot d^3}{32}.$$

2. Verification of shafts for fatigue. This is done in areas with stress concentrators such as:

- ;
- ;
- ;

Note: At each rotation by 180° the fibers are compressed and then stretched, so the loading is alternating symmetric (III), and the torsional loading is pulsating (II).

Fatigue verification involves determining the *safety factor* and comparing it with the *permissible value*.

$$c = \frac{1}{\sqrt{\frac{1}{c_\sigma^2} + \frac{1}{c_\tau^2}}},$$

$$c_\sigma = \frac{1}{\frac{\beta_{k\sigma}}{\varepsilon_\sigma \cdot \gamma_\sigma} \cdot \frac{\sigma_v}{\sigma_{-1}} + \frac{\sigma_m}{\sigma_{02}}} \quad c_\tau = \frac{1}{\frac{\beta_{k\tau}}{\varepsilon_\tau \cdot \gamma_\tau} \cdot \frac{\tau_v}{\tau_{-1}} + \frac{\tau_m}{\tau_{02}}}$$

where :

c_σ - ;

c_τ - ;

β_k - ;

ε - ;

γ - ;

σ_{-1} - ;

σ_{02} - ;

$$\text{and } \sigma_v = \sigma_{i \max}, \quad \tau_v = \tau_m = \frac{\tau_{t \max}}{2}, \quad \sigma_m = 0.$$

The specific values of the existing factors can be found in the specialized literature.

3. Verification of shafts for deformations and vibrations.

The bending deformation is given by the deformed middle fiber, which determines the magnitude of the deflection f and the angle of inclination α . When forces act in different planes, the final elastic curve and the angular deviation resulting from the supports are obtained by the geometric summation of the deformations in the planes where the work was done.

Depending on the nature and direction of the disturbing loads, straight shafts may experience during operation:

- ;
- ;
- ;
- ;

Usually, only the first two types of vibrations are significant for straight shafts, as they can become dangerous.

[illegible]

11. COUPLINGS

The couplings are.....
.....
.....
.....

These are used.....
.....
.....

Figure 11.1 shows the kinematic diagram of the coupling connection between two shafts.

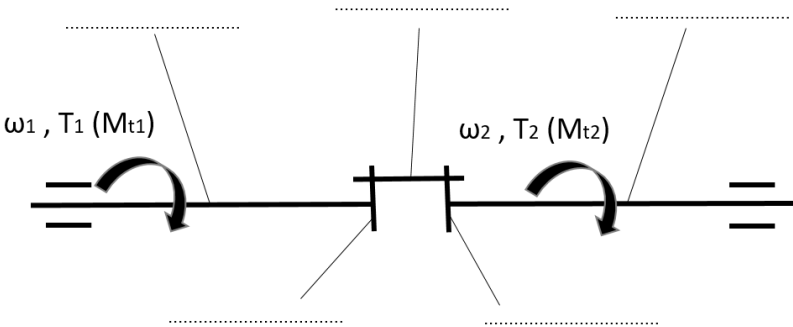


Fig. 11.1 Kinematic diagram of coupling connection

Objectives pursued through the use of couplings.

- a)
.....
.....
.....

b)

.....

.....

.....

c)

.....

.....

.....

d)

.....

.....

.....

e)

.....

.....

.....

Classification.

Mechanical couplings:

1. Permanent

a) Fixed: - ;

- ;

- ;

- ;

- ;

- b) Movable: - ;
 -

2. Intermittent:

a) Based on the way torque transmission is:

- ;
 - ;
 - ;

b) Based on the nature of operation:

- ;
 - ;
 - ;
 - ;
 - ;
 -

The following loads act on the elements of a coupling:

- ;
 - ;
 - ;

The coupling is calculated for a design torsion torque:

$$M_{tc} = k \cdot M_t,$$

where: M_{tc} : ;

M_t : ;

k – : ;

Permanent fixed couplings

Flanged coupling (Figure 11.2).

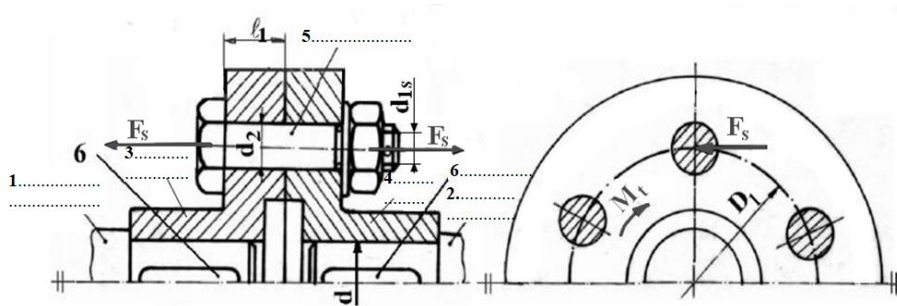


Fig. 11.2 Couplings with flange

They are made up of:

.....

Halfcouplings.....

.....

Torsion torque.....

.....

The centering.....

The torsion torque that can be transmitted can be determined using the formula:

$$M_{tc} = M_{tf} = \mu \cdot F_s \cdot z \cdot \frac{D_1}{2},$$

where:

M_{tf} -;

μ -;

F_s -;

z -;

D_1 -

Since the bolts are tightly fastened during assembly, they are subjected to both tensile and torsion, therefore:

$$\beta \cdot F_s = \frac{\pi \cdot d_{1s}^2}{4} \cdot \sigma_t,$$

where:

β -;

d_{1s} -;

σ_t -

In this way, the sizing of the bolt is done using the formula (for non-standardized couplings):

$$d_{1s} = \sqrt{\frac{8 \cdot \beta \cdot M_{tc}}{\mu \cdot \pi \cdot z \cdot D_1 \cdot \sigma_t}}.$$

The transmissible torsion torque will be corrected as follows:

$$M_{tc} = C_s \cdot M_t,$$

where: C_s -

and takes into account..... ;

In the case of standardized couplings (ISO 14691), with bolts of known dimensions, only their checking is performed.

For those installed with no clearance, checking is done for....., and..... .

$$\sigma_{total} = \frac{8\beta M_{tc}}{\pi \mu z D_1 d_{1s}^2} \leq \sigma_{ai}$$

$$\tau_f = \frac{8 \cdot M_{tc}}{z \cdot \pi \cdot D_1 \cdot d_2^2} \leq \tau_{af}$$

where: F_{tc} - ;

d_2 -

In the case of flanged couplings mounted with bolts with clearance (commonly encountered in practice), the alignment of the half-couplings is done using spacers or rings (Figure 11.3), and the bolts will be sized using the formula for d_{1s} and will be checked for..... .

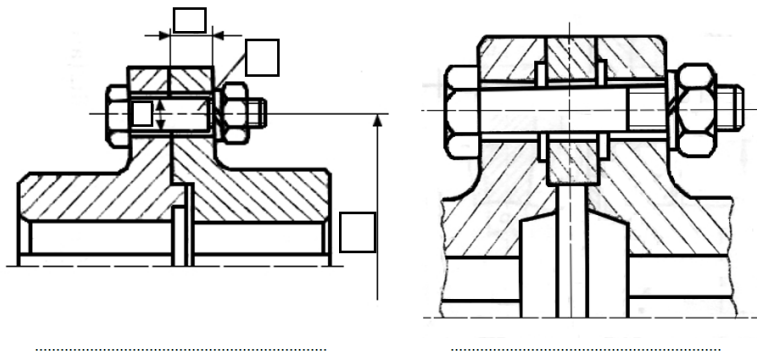


Fig. 11.3 Examples of couplings mounted with clearance and types of centering

Verification for bending is done using the formula:

$$\sigma_i = \frac{32 \cdot M_{tc} \cdot l_1}{\pi \cdot D_1 \cdot d_2^3} \leq \sigma_{ai},$$

These couplings are used for:

- ;
- ;
- ;

Permanent movable couplings

Couplings with rigid elements for angular misalignments (Figure 11.4).

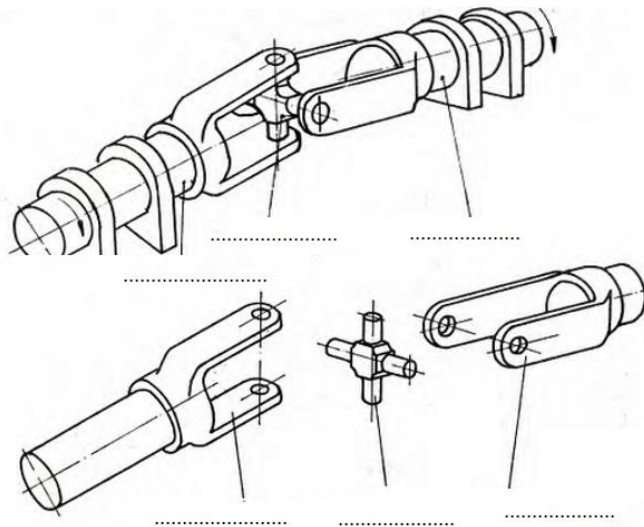


Fig. 11.4 Cardan joint

The cardan joints (couplings) allows for.....

.....

.....

It is composed.....

.....

.....

The forks are offset by..... .

It is frequently used..... .

Forces and torques calculation.

Angular velocity of the driven shaft ω_2 varies over time even if $\omega_1 = \text{constant}$ (Figure 11.5), between a minimum value (when the cross is in a perpendicular plane to the motor shaft) and a maximum value (when the cross is in a perpendicular plane to the driven shaft).

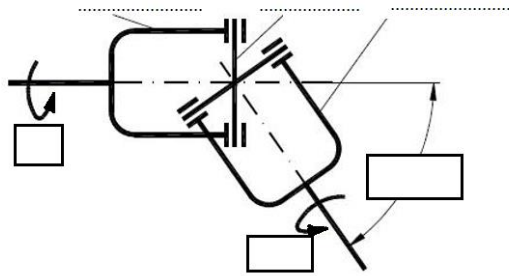


Fig. 11.5 Cardanic joint kinematics

$$\frac{\omega_{2\max}}{\omega_{2\min}} = \frac{1}{\cos^2 \alpha}$$

The shaft 1 transmits a torsion torque M_{t1} . The spindles connected to the fork of shaft 1 are loaded by the forces F_1 , which are null for $\phi_1=0$ and have maximum values for $\phi_1=90^\circ$.

$$F_1 = \frac{M_{t1}}{d_c} ; \quad F_{a1} = F_1 \cdot \tan \alpha ; \quad M_{t2\max} = M_{t1} \cdot \frac{\omega_1}{\omega_{2\min}} = \frac{M_{t1}}{\cos \alpha} ;$$

$$F_{2\max} = \frac{M_{t2\max}}{d_c} = \frac{M_{t1}}{d_c \cdot \cos \alpha} ; \quad F_{a2} = F_1 \cdot \sin \alpha$$

where: $F_{2\max}$ - ;
 F_{a2} -

Because ω_2 varies, to ensure the constancy of this speed, so to achieve a synchronous construction, two cardan joints can be connected, so that:

- ;
- ;
-

The materials for the cardan coupling elements are:

.....

Couplings with elastic elements (Figure 11.6)

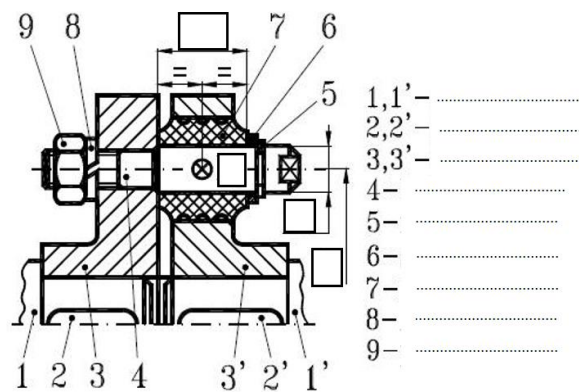


Fig. 11.6 Elastic coupling with bolts

Elastic couplings are characterized by:.....

.....

Goals pursued:

-

..... ;

- ;

-

The stiffness of the coupling is.....

.....

..... .

The characteristic of a coupling is.....

.....

..... .

Advantages:

-
..... ;

-
..... ;

-
..... ;

-
..... ;

-
..... ;

-
..... ;

-
..... ;

-
..... ;

-
..... ;

Materials used for coupling elements:

-
..... ;

-
..... ;

The choice of a coupling of this type is made by determining the torsion torque:

$$M_{tc} = C_1 \cdot M_t$$

Checks are only made in certain special situations, and:

$$M_t = F_{t1} \cdot \frac{D_1}{2} \cdot z, \text{ where: } F_{t1} = \frac{2 \cdot M_t}{D_1 \cdot z}.$$

Checking the bolt-bush crushing:

$$\sigma_s = \frac{F_{t1}}{d_b \cdot l} \leq \sigma_{sa},$$

where: d_b :

l :

Checking at bending:

$$\sigma_i = \frac{M_i}{W_i} = \frac{32 \cdot F_{t1} \cdot l}{\pi \cdot d_b^3}.$$

Figure 11.7 shows some examples of coupling types.: a) With flanges;
b) Cardan joint; c) With bolts.



(a)....., (b)....., (c).....

Fig. 11.7 Coupling examples

[illegible]

12. ROLLING BEARINGS. BEARINGS

Bearings are.....

.....
.....
.....

The component elements are (Figure12.1):

- ;
- ;
- ;
-

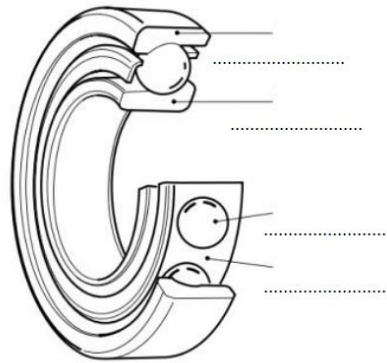


Fig. 12.1 A bearing component elements

Observation:.....

.....
.....

Advantages	Disadvantages
.....	
.....	
.....	
.....	
.....	
.....	

Bearings classification and simolyzation (SR ISO 15:2017, ISO 492, ISO 582, ISO 355)

- By the shape of the rolling elements:

..... ;

..... ;

..... ;

..... ;

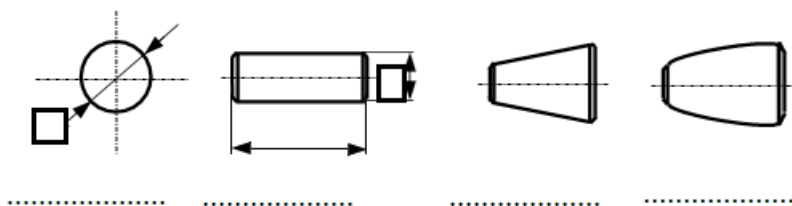


Fig. 12.2 Types of rolling elements

- According to the direction of the main force taken:

..... ;
 ;

- By the number of rows in which the rolling elements are arranged:

..... ;

- According to the adaptability of the inner ring to the spindle direction:

..... ;

- By the value of the radial clearance:

..... ;
 ;

- According to the overall dimensions in the radial direction:

.....

- According to the overall dimensions in the axial direction:

.....

The size series are shown in the Figure12.3.

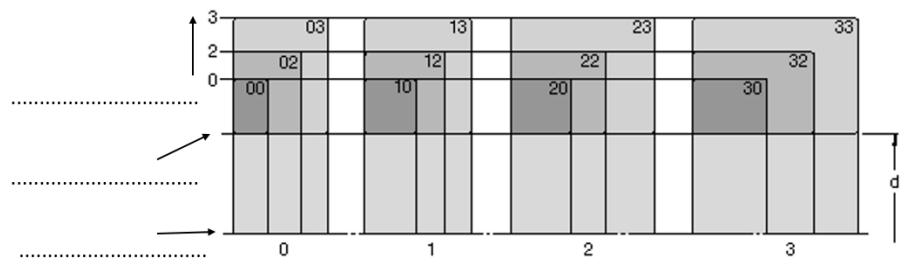


Fig. 12.3 Size series

Bearings symbolization

It consists of a basic symbol and an additional one.

The basic symbol consists of two groups of letters or numbers:

1.First group.....
.....
.....

2.Second group.....
.....
.....
.....

The additional symbol is used for bearings that have a modified construction and consists of:

-
..... ;
-
..... ;
.....

The bearing symbolization scheme (abcd or dabc) is shown in Figure 12.4. Interpretation of the symbol:

a) The symbol for the bearing type (radial, tapered roller, thrust, etc.) is represented by..... ;

b) The symbol of the size series represented by two digits:

First digit..... ;

Second digit.....

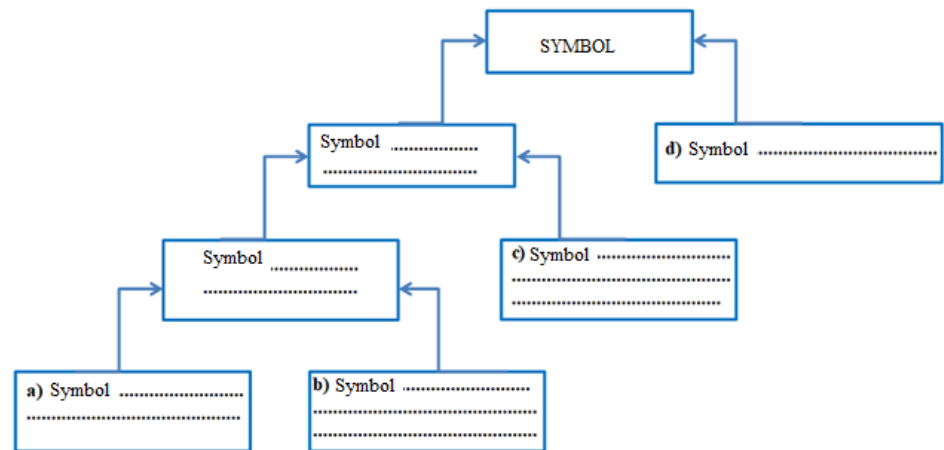


Fig. 12.4 Bearing symbolization scheme

c) The symbol for the inner diameter, thus:

For a diameter up to 9 mm and over 480 mm there is a digit(s) indicating..... .

The symbol for an inner diameter between 10 and 17 mm is represented as follows:

Inner diameter (mm) 10 12 15 17

Inner diameter symbol 00 01 02 03 .

The symbol for an inner diameter between 20 and 480 mm is indicated by two digits, a number obtained by.....
..... .

d) The extra symbol.

Bearing calculation elements

Design of roller bearings.

1. by dynamic load capacity.....

.....

2. by static load capacity.....

.....

The service life of rotating bearings is limited.....
on the raceways of the rings or the rolling elements, as a result of fatigue of
the superficial layers of the material.

To avoid this form of damage, rolling bearings are designed for durability.

To perform the bearing calculation, it is necessary to define some specific
concepts:

1. The bearing durability L :

.....

.....

2. Durability of a batch of identical bearings L_{10} :.....

.....

.....

.....

.....

.....

.....

.....

Basic durability.....

.....

.....

.....

3. Basic dynamic load capacity C :

.....

.....

.....

.....

Between the dynamic load capacity C , the load P that loads the bearing, and its durability L , the following experimental dependence was established, which represents the equation of the fatigue curve::

$$P \cdot L_{10}^{1/p} = C ,$$

where: p -

.....

The calculation of bearings can be done in the following way:

a) if known the load on the bearing P , the speed of the rotating ring n and the required operating time are known $L_{h \text{ required}}$ (operation hours), than the bearing is calculated according to the *dynamic load capacity*.

$$C_{\text{req}} = P \cdot \sqrt[p]{L} \leq C_{\text{catalogue}} , \text{ where}$$

$$L = \frac{60 \cdot n \cdot L_{h \text{ required}}}{10^6} \text{ [million of rotations]} .$$

b) for an existing bearing, with dynamic load capacity $C = C_{\text{catalogue}}$, whose rotating ring has a speed n and which is loaded with a load P , the calculation is carried out according to the durability criterion, with the relations:

$$L = \left(\frac{C_{\text{catalogue}}}{P} \right)^p, \quad L_h = \frac{10^6 \cdot L}{60 \cdot n} \geq L_{h \text{ required}}.$$

4. Equivalent dynamic load P.

In the case of bearings where combined forces act, the radial forces and an equivalent dynamic load P replaces F_r and thrust forces F_a . This is

.....

To calculate the actual value of any acting force, a global correction factor will be introduced, which takes into account the specific operating conditions of the bearing.

$$P_c = f \cdot P; \text{ and } f = f_z \cdot f_d \cdot f_v \cdot f_s \cdot f_t, \text{ where:}$$

f_z - ;

f_d - ;

..... ;

f_v - ;

f_s - ;

f_t -

The experimental diagram in Figure 12.5 represents the correlation between the radial force that load the bearing —.....
 F_r — and the thrust force
 F_a , for an equivalent dynamic load
 P constant.

In the diagram, two areas appear, delimited by a straight line inclined at an angle $\beta_2 = \arctg e$, where e is a bearing constant, the values of which are given in the bearings catalogue.

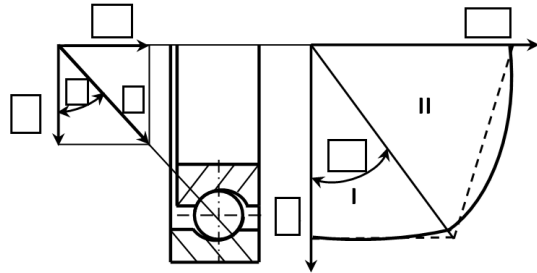


Fig. 12.5 Bearing force correlations

Area I is characterized by.....

.....

$$\frac{F_a}{F_r} = \tg \beta \leq \tg \beta_2 = e,$$

the equivalent dynamic load is calculated with the relationship:

$$P = f \cdot V \cdot F_r,$$

Area II is characterized by

.....

$$\frac{F_a}{F_r} = \tg \beta > \tg \beta_2 = e,$$

the equivalent dynamic load is calculated with the relationship:

$$P = f \cdot (V \cdot X \cdot F_r + Y \cdot F_a), \text{ where:}$$

β -

X and Y -

.....

V -

$f - \dots\dots\dots$;

$V = 1 \dots\dots\dots$;

$V = 1,2 \dots\dots\dots$.

Functions of roller bearings

The function of first-class (I).

1. Supporting and centering the shafts.

- Supporting.....

.....

- Centering.....

.....

.....

2. Axial shaft positioning.....

.....

.....

-

.....

.....

-

.....

.....

3. Ensuring free expansion of the shaft at operating temperature.....

.....

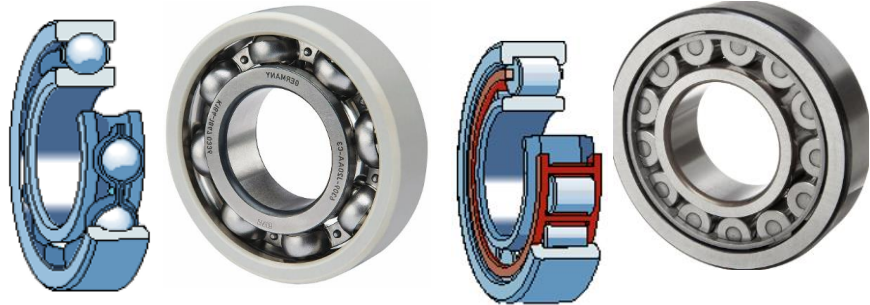
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.....

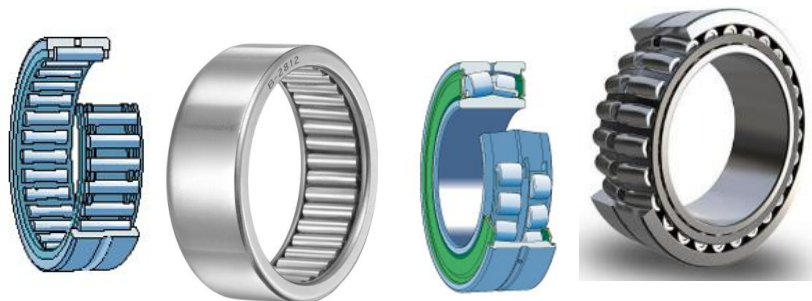
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Figure 12.6 illustrates different types of bearings: a) radial ball bearing; b) radial roller bearing; c) needle roller bearing; d) spherical roller bearing; e) angular contact ball bearing; f) tapered roller bearing; g) thrust ball bearing; h) double direction thrust ball bearing.



(a).....,

(b).....,



(c).....,

(d).....

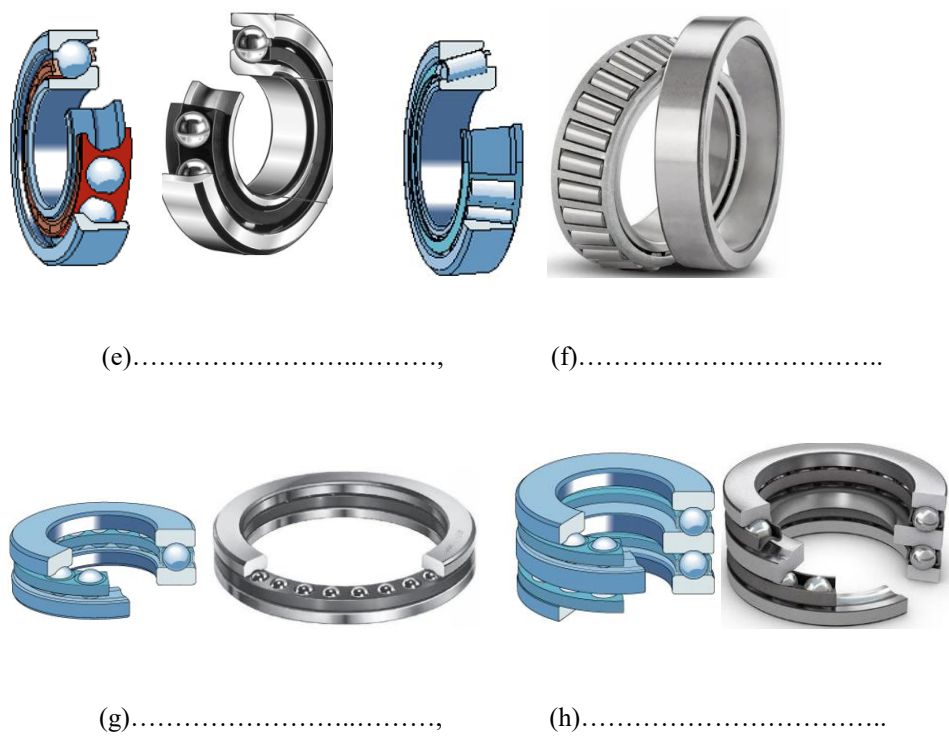


Fig. 12.6 Bearings examples

Notes / Observations

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